Semiconductor Lasers

High-speed & Nonlinear Dynamics

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Textbooks

[1] J. M. Liu, *Photonic Devices*, Cambridge University Press

[2] L. A. Coldren & S. W. Corzine, *Diode Lasers and Photonic Integrated Circuits*, Wiley

[3] P. Blood, *Quantum Confined Laser Devices*, Oxford University Press

[4] J. Faist, *Quantum Cascade Lasers*, Oxford University Press

[5] J. Ohtsubo, *Semicondutor Lasers*, Springer

[6] T. Erneux & P. Glorieux, *Laser Dynamics*, Cambridge

[7] A. Uchida, *Optical Communications with Chaotic Lasers*, Wiley

Let there be light…

LASER: Light Amplification by Stimulated Emission of Radiation

Laser light is monochromatic, i.e., consisting of a single wavelength or color, and emitted in a narrow beam

T. H. Maiman was an American engineer and physicist credited with the invention of the first working laser. The laser was successfully fired on May 16, 1960

Theodore Maiman (1927-2007)

The laser disc player (1978), was the first successful consumer product to include a laser, but the compact disc player was the first laser-equipped device to become truly common in consumers' homes, beginning in 1982, followed shortly by laser printers

T. H. Maiman, Journal of the Optical Society of America, vol. 50, pp. 1134, (1960)

…and more!

Goldfinger (1964) was the first film to feature a laser

Lasers did not exist in 1959 when the book was written. In the novel, Goldfinger uses a circular saw to try to kill Bond, but the filmmakers changed it to a laser to make the film feel more fresh

High-speed lasers

Modulation dynamics

Outline

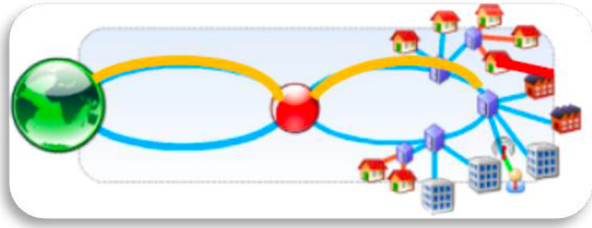
- Introduction to optical communications
- High speed lasers
- Semiconductor lasers: Basic principles
- Corpuscular equations
- Modulation dynamics

Optical communications

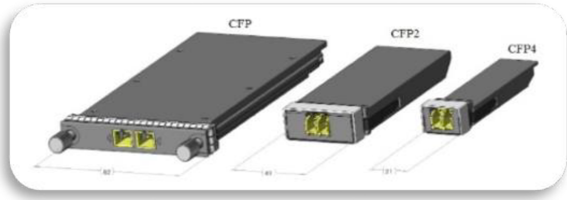
TELECOM

> 10 km

- ➢Metro & Access



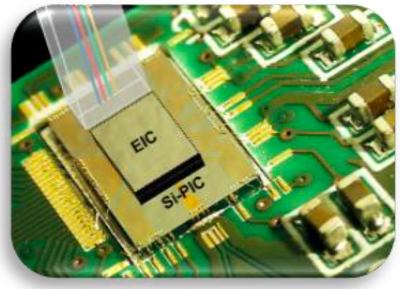
- **1.5 µm**
- ➢ Transceivers modules



DATACOM

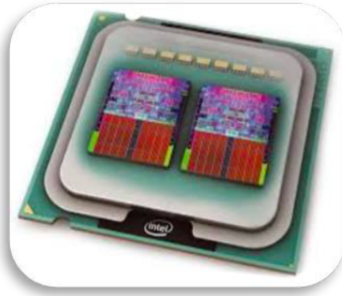
- **1m – 10 km**
- ➢ Routers & switches

- **1.3 µm**
- ➢ Rack-to-rack
- ➢ Board-to-board

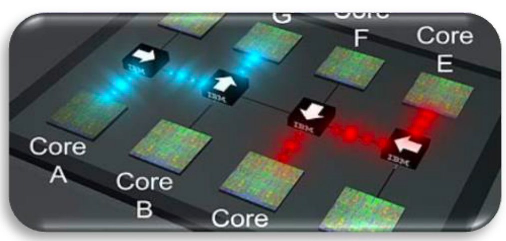


COMPUTERCOM

- **< 1m**
- ➢ Photonics integrated circuits

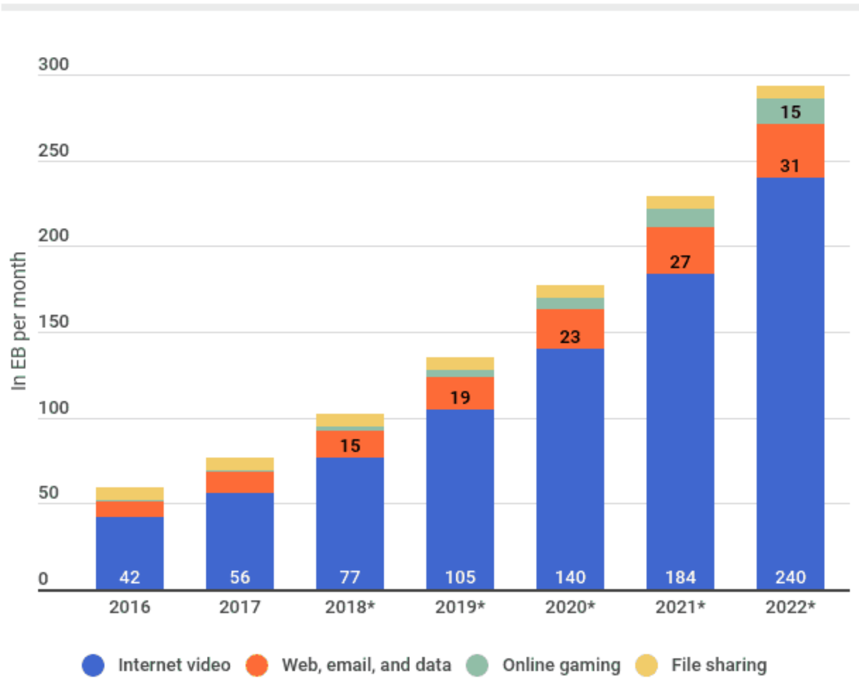


- **1.3 µm**
- ➢ Transceivers on chip



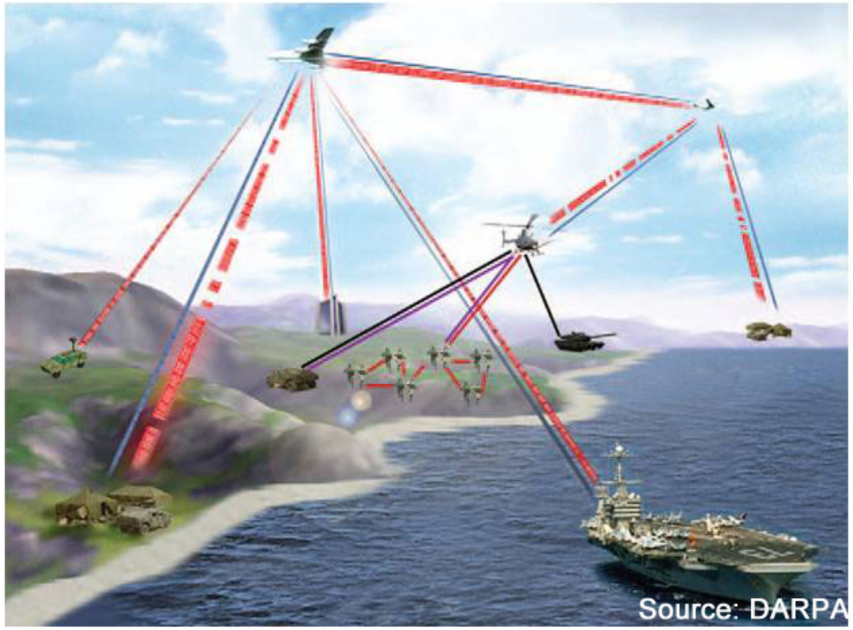
Optical communications

Predicted worldwide internet traffic (1EB = 10^{18})



Sources: Cisco Systems

Created by Dailywireless.org



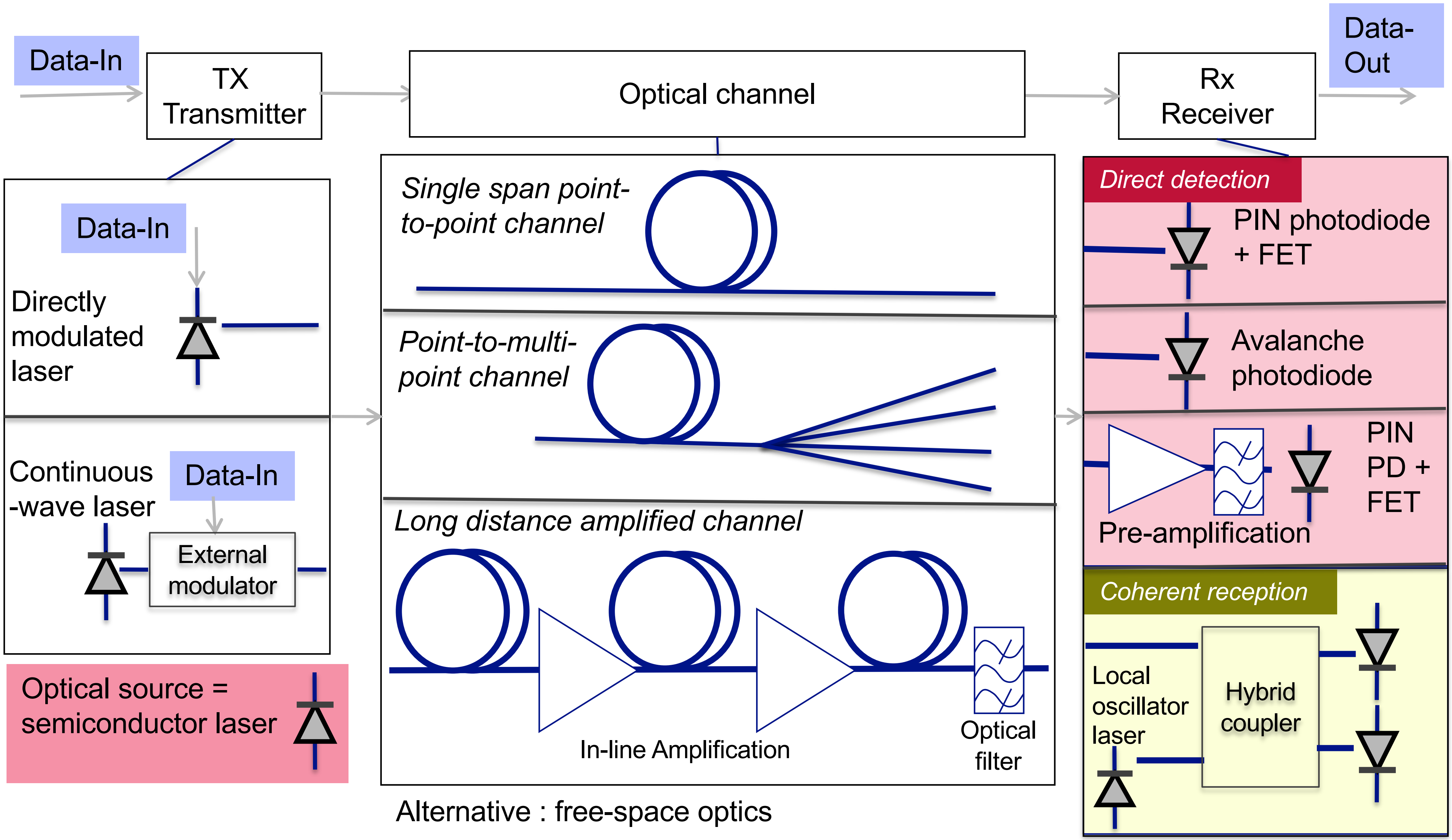
Optical link

- → 1000 x Larger bandwidth
- → 1000 x Lower loss
- → 100 x Larger distance

Scalability & power efficiency

Free-space optical communications ramping up

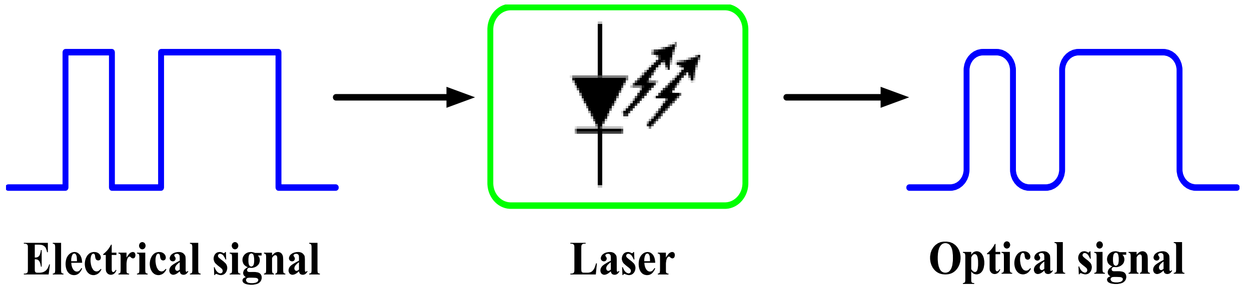
Architecture of an optical link



Alternative : free-space optics

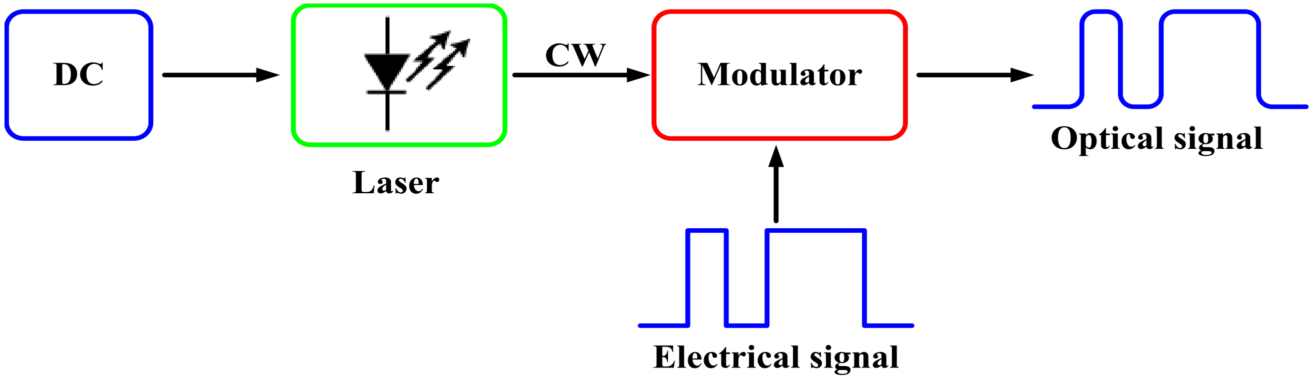
Modulation schemes

Direct modulation



- Chirp
- Simple
- Compactness
- Low cost
- Energy efficient

External modulation

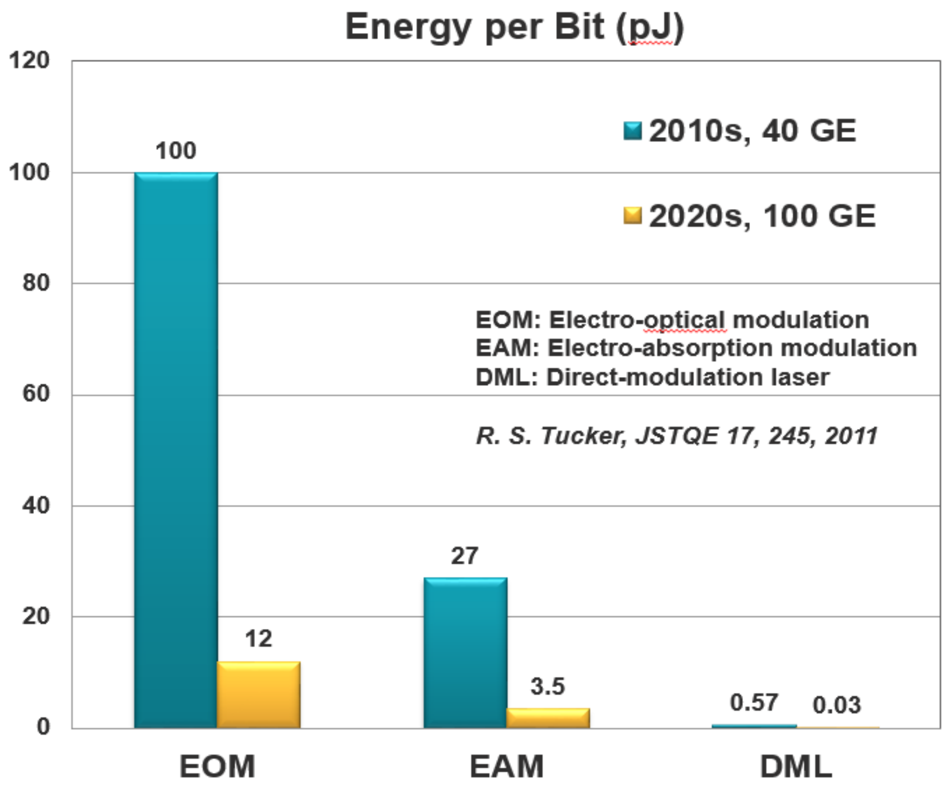


- Power consumption
- Cost
- High-speed
- Zéro chirp

T. Anfray et al., Journal of Lightwave Technology, vol. 30, pp. 3089, (2012)

Energy consumption

Transmitter Type	Component		Energy per Bit: 2010-era technology (40 Gb/s)	Energy per Bit: 2020-era Target (100 Gb/s)
	MUX		10 pJ	2 pJ
Directly Modulated	Laser $(\overline{V}_{laser}\overline{I}_{laser})$		358 fJ	10 fJ
	Driver (P_{driver}/B_r)		212 fJ	20 fJ
	Laser + Driver		570 fJ	30 fJ
Externally Modulated (Electro-Absorption)	Laser $(\overline{V}_{laser}\overline{I}_{laser}/B_r)$		1.5 pJ	500 fJ
	Lumped $V_{mod} = 3$ V	$\frac{1}{2}C_{mod}(V_{mod}^{p-p})^2$	800 fJ	50 fJ
		$2\hat{V}_{mod}^2/Z_0B_r$	4 pJ	1.6 pJ
		$\overline{V_{mod}I_{mod}}/B_r$	1.5 pJ	600 fJ
	TW $V_{mod} = 1$ V	$2\hat{V}_{mod}^2/Z_0B_r$	440 fJ	176 fJ
		$\overline{V_{mod}I_{mod}}/B_r$	500 fJ	200 fJ
	Driver (P_{driver}/B_r)		25 pJ	100 fJ (C_{mod}) 3 pJ (Z_0)
	Laser + Driver		27 pJ	600 fJ (C_{mod}) 3.5 pJ (Z_0)
Externally Modulated (Electro-Optic)	Laser $(\overline{V}_{laser}\overline{I}_{laser}/B_r)$		1.5 pJ	500 fJ
	$2\hat{V}_{mod}^2/Z_0B_r$		4 pJ	1.6 pJ
	Driver (P_{driver}/B_r)		25 pJ	3 pJ
	Laser + 4 Drivers		~ 100 pJ	12 pJ



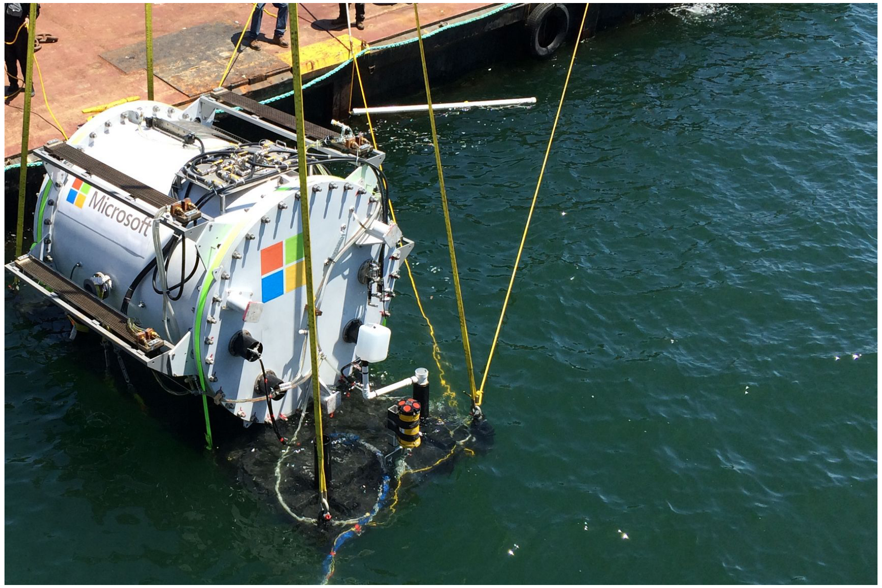
R. S. Tucker, IEEE J. Sel. Top. Quantum Electron. 17, pp. 245 (2011)

Energy consumption

An optical datacenter burns about 1000 TWh

Air cooling represents 50% of the total energy consumption ie. 1kW used for running a server needs an additional 1kW to maintain to cool it down that is to say to maintain its temperature to 22°C

- ***→ Microsoft : under the sea data centers powered by renewable energy***
- ***→ Facebook: datacenters near the edge of the Artic circle***

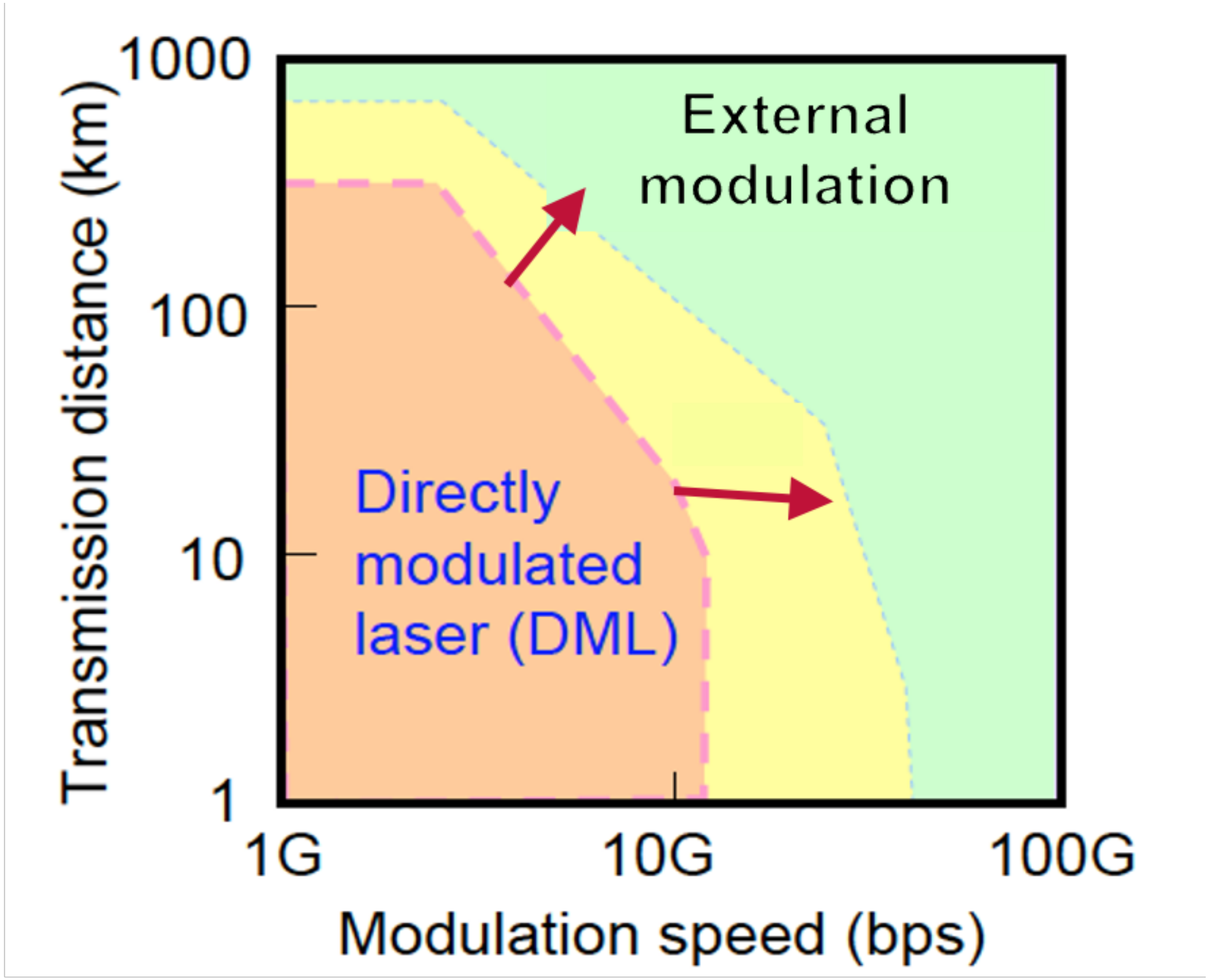


To reduce the global energy consumption, data centers with energy efficient optical components and using renewable energies is required

Speed & range limitations

Tradeoff between the modulation speed and the transmission distance

Beyond a certain range, external modulation is mandatory (core networks)

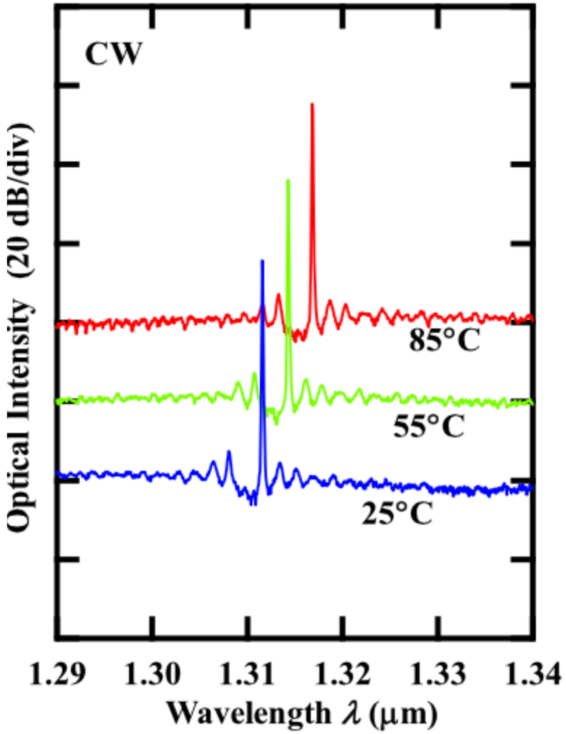


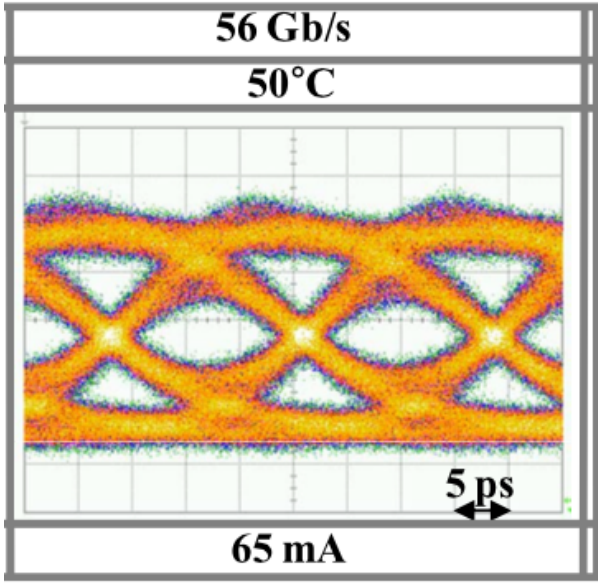
High-speed lasers

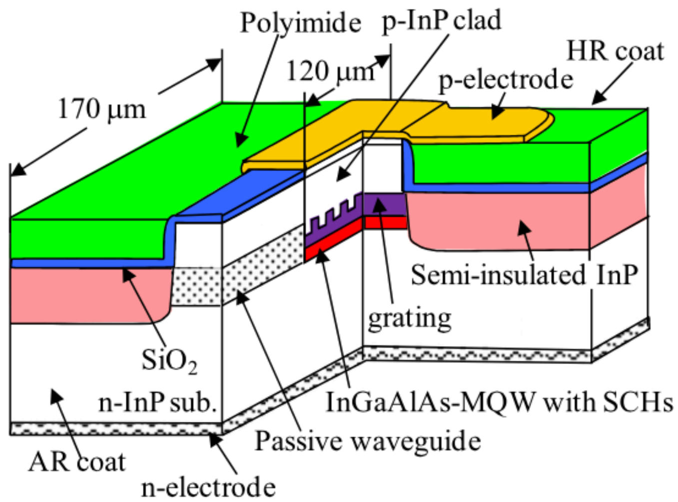
OCLARO

HITACHI

Direct modulation@56 and 50 Gb/s w/ 1.3-µm InGaAlAs DFB lasers
Transmission over a 10 km SMF



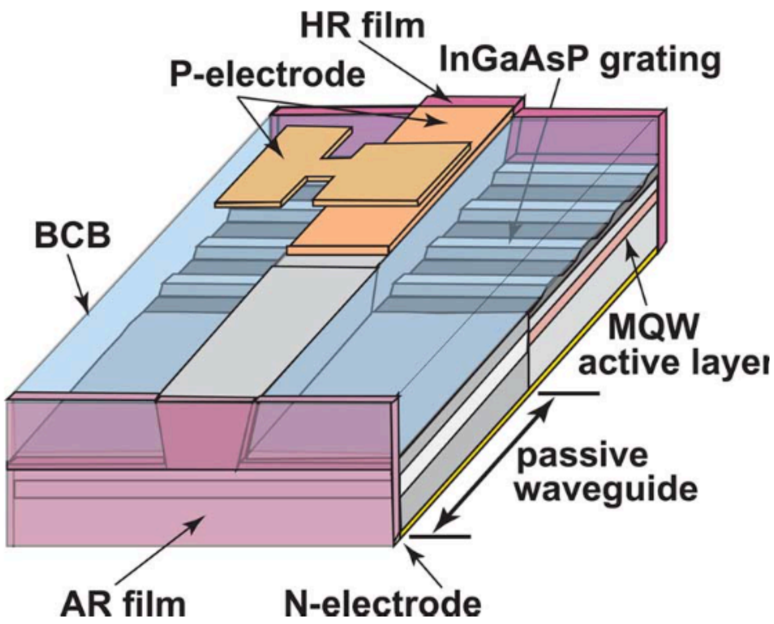




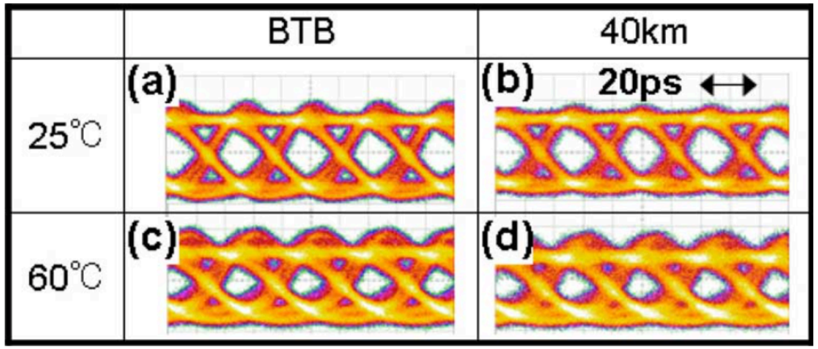
K. Nakahara, IEEE Photonics Technology Letters, vol. 27, pp. 534-537, (2015)

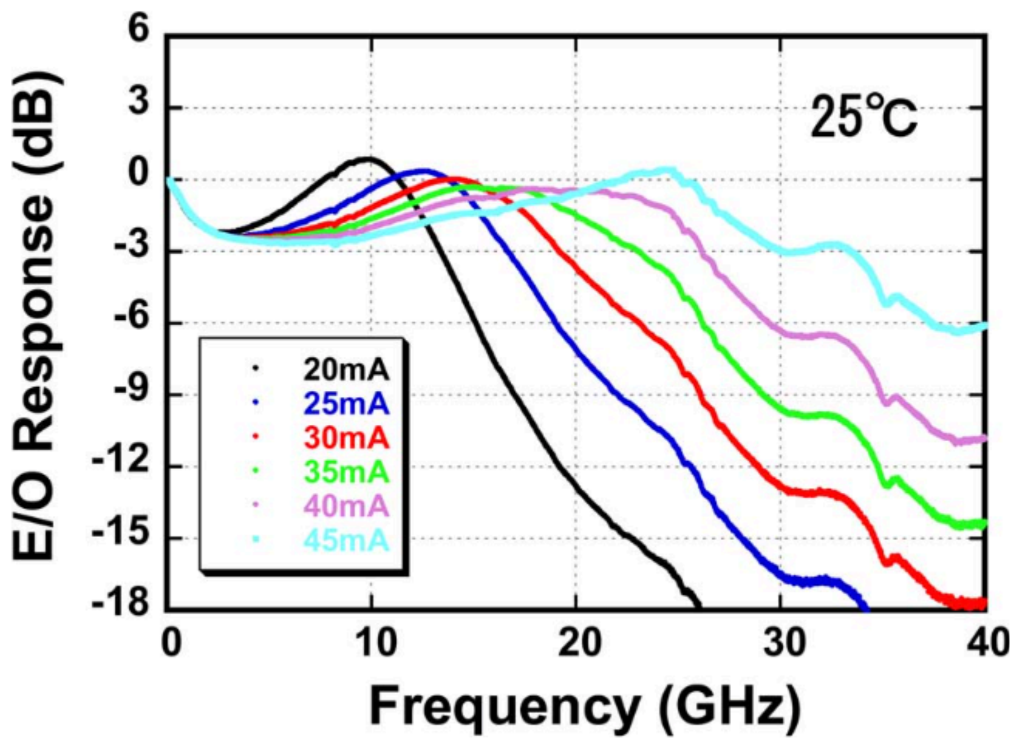
High-speed lasers

NTT Electronics



1.3-µm InGaAlAs DFB lasers
3-dB bandwidth of 30 GHz
Clear eye @43 Gb/s after 40 km SMF transmission under direct modulation

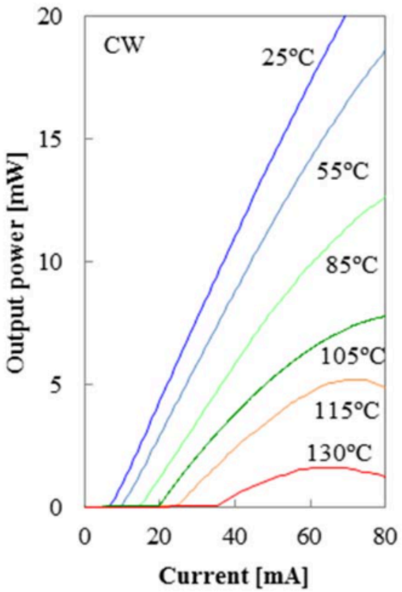


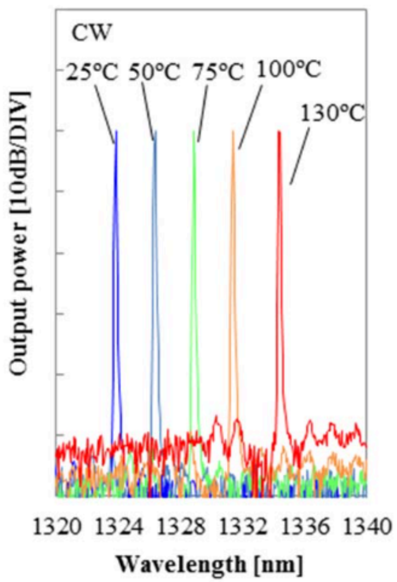


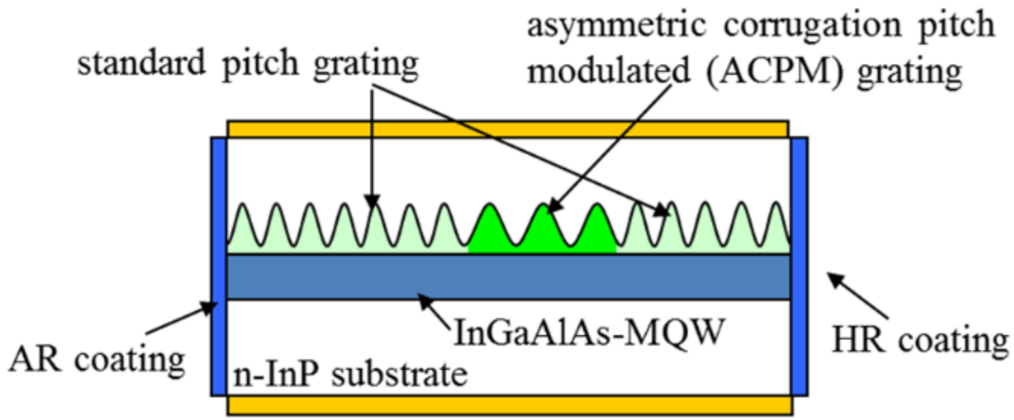
T. Tadokoro, IEEE J. of Lightwave Technology, vol. 30, pp. 2520-2524, (2012)

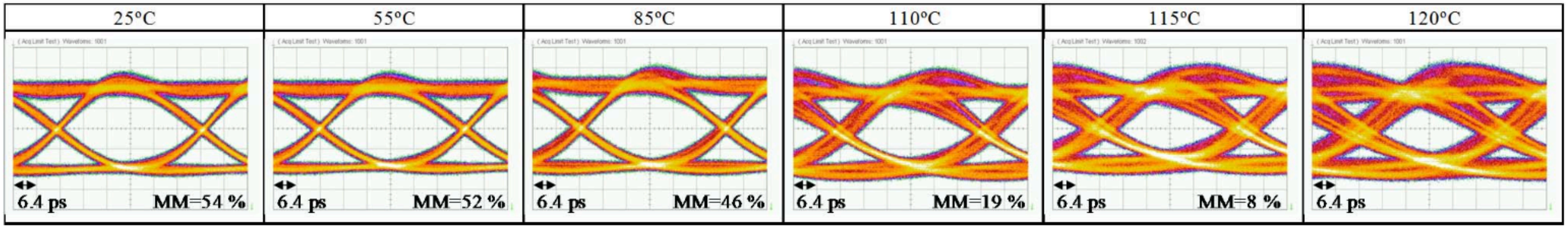
High-speed lasers

High-temperature 25.8-Gbps clear-eye-opening of a 1.3-µm directly modulated InGaAlAs-MQW DFB laser up to 120C







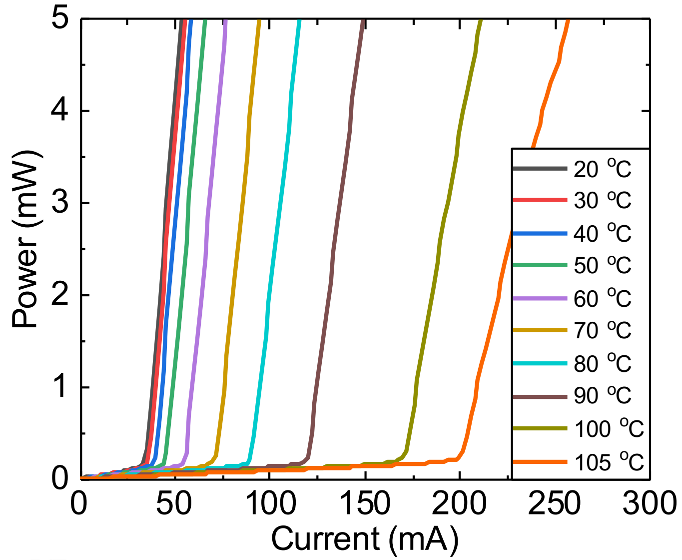


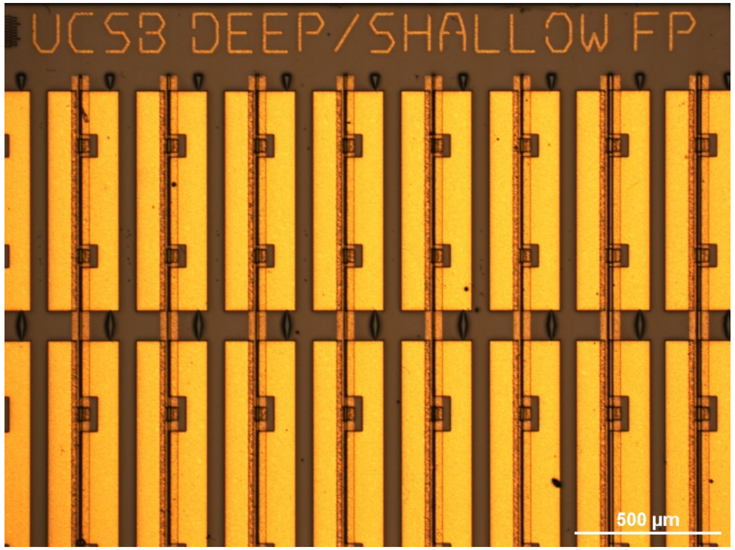
A. Nakanishi et al. , IEEE International Semiconductor Laser Conference, paper WA3, (2016)

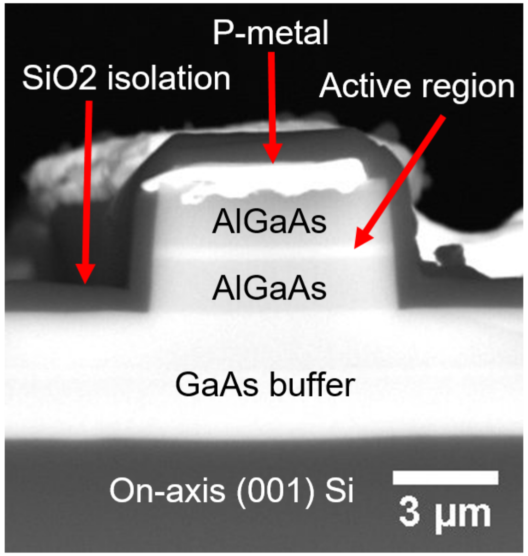
High-speed lasers

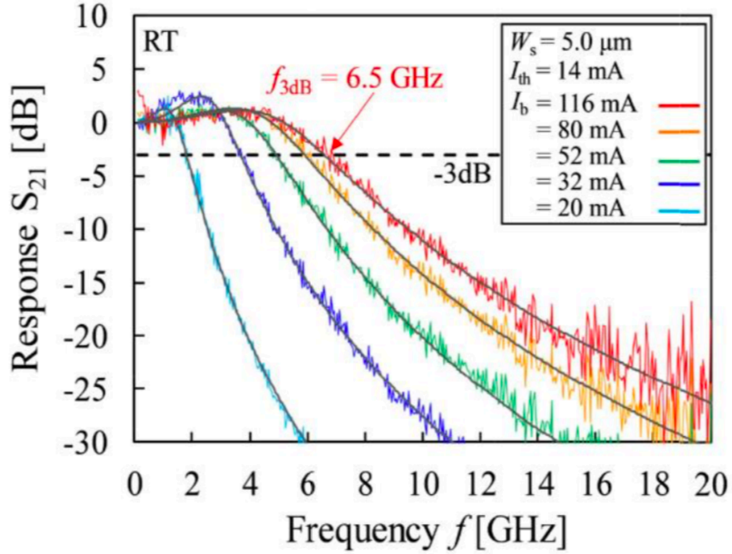
High performance lasers on silicon

- → Low dislocation density
- → CW threshold current of ~5 mA
- → Wall plug efficiency of ~40%
- → Single-facet output power 176 mW
- → Continuous wave operation >100°C







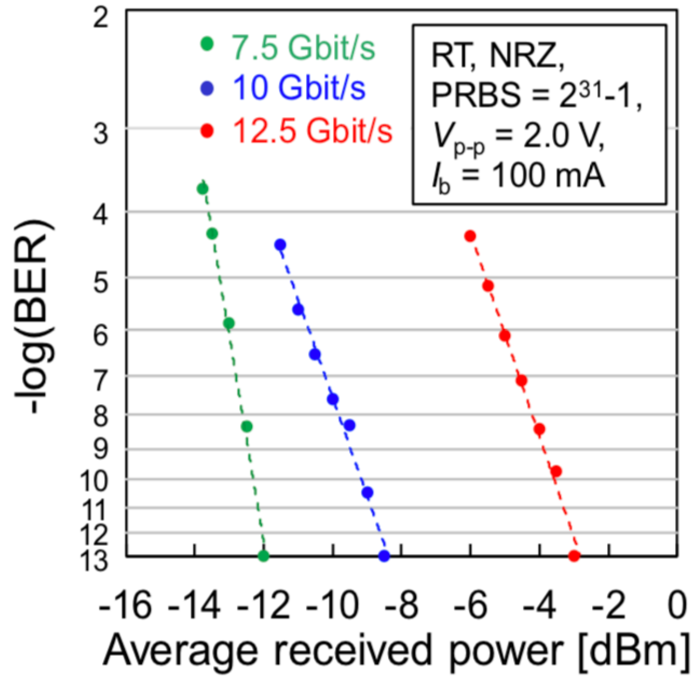


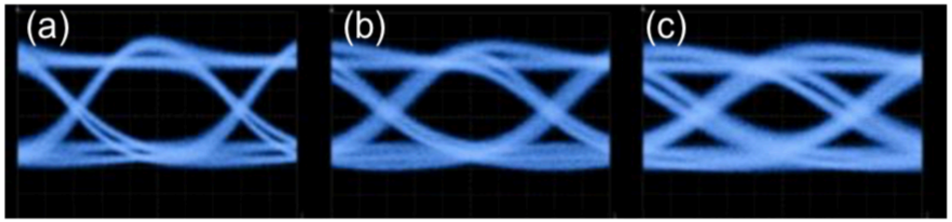
J. Norman et al. APL Photonics (2018)

High-speed lasers

High performance lasers on silicon

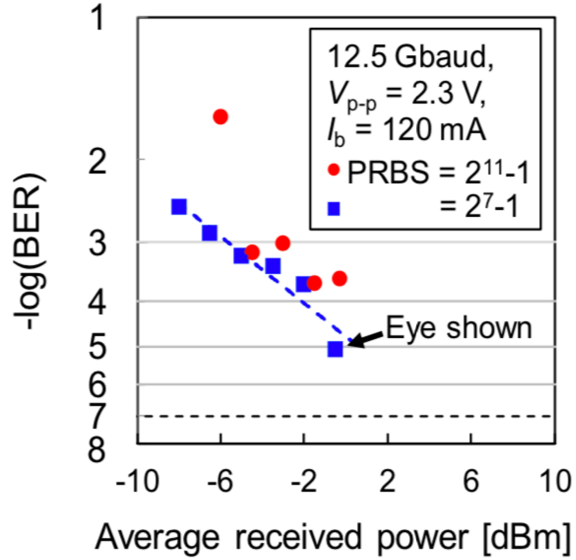
OOK



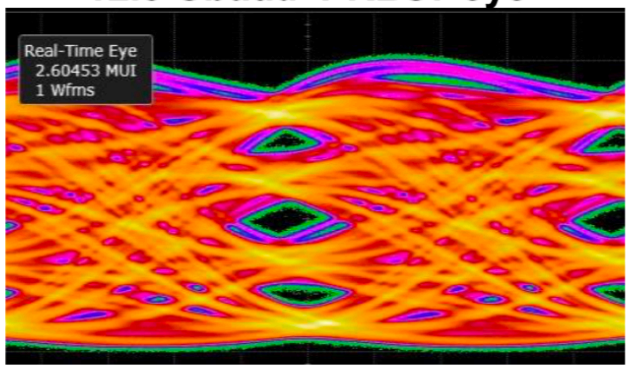


J. Norman et al. ECOC (2018)

PAM4

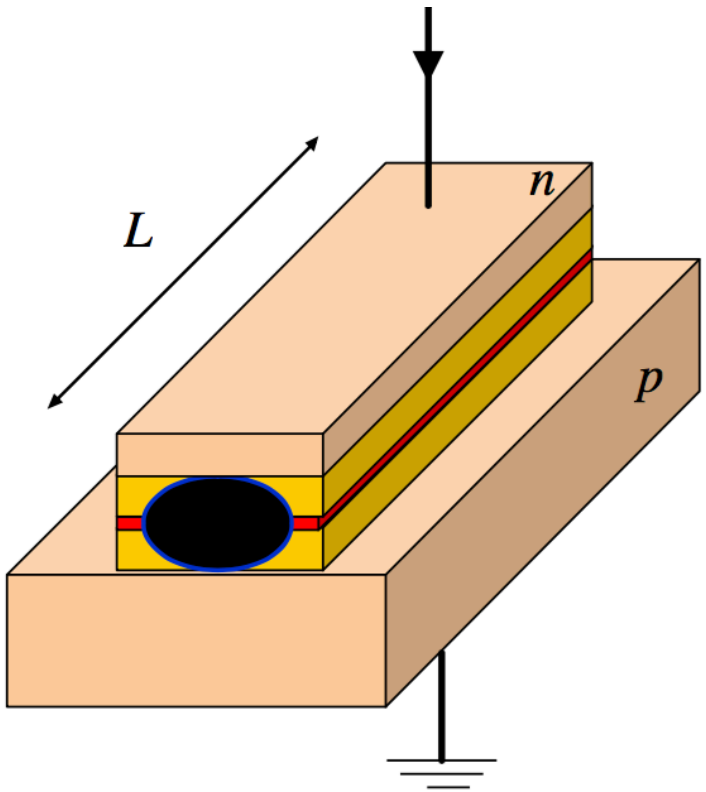


12.5 Gbaud PRBS7 eye

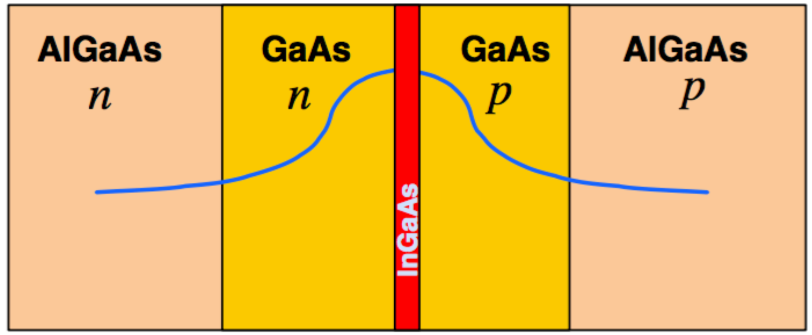


Semiconductor lasers

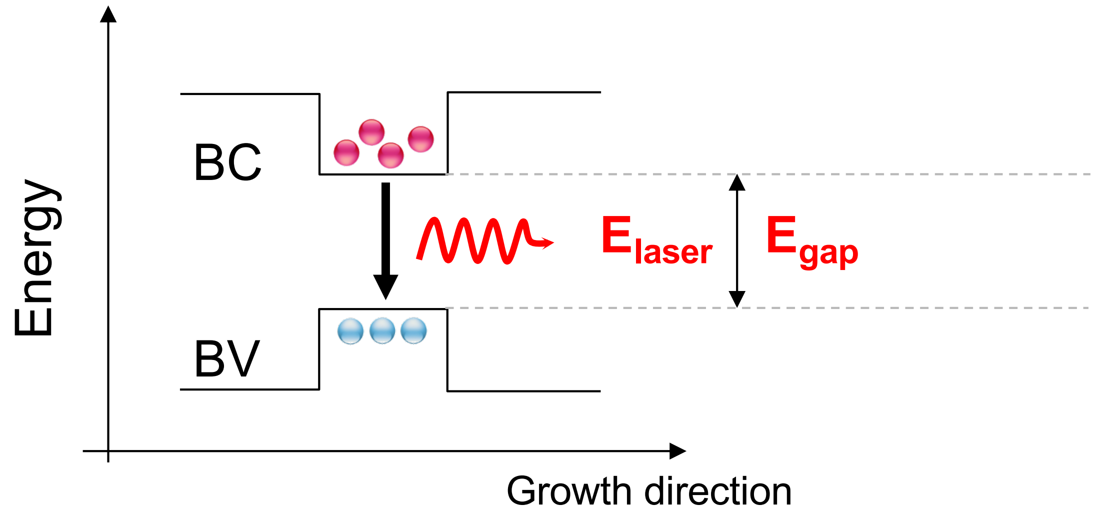
3 equivalent aspects



Technology

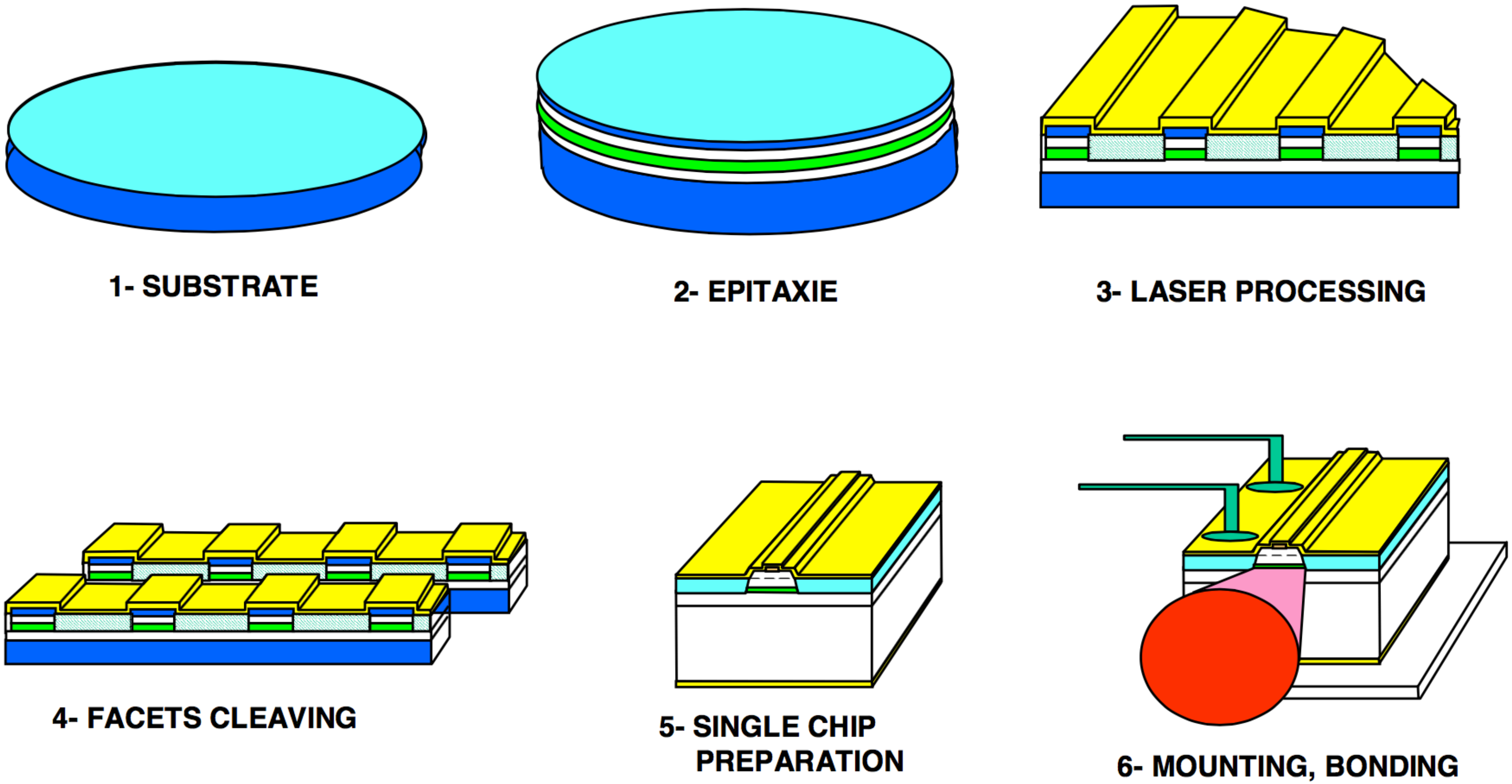


Electromagnetism

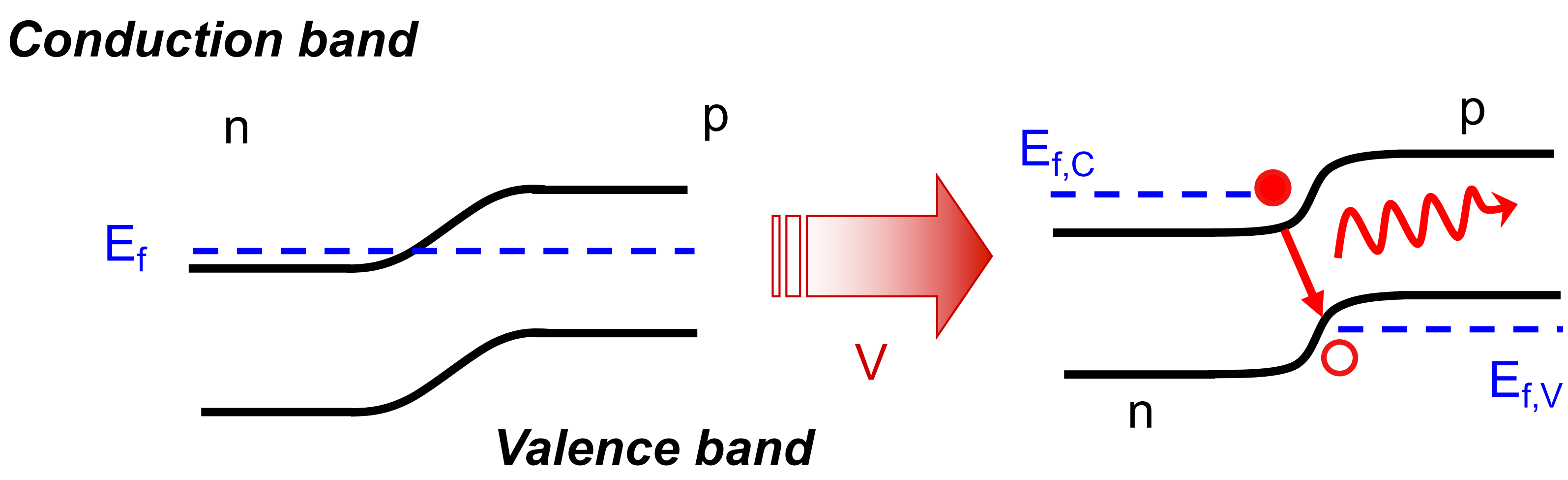


Interband structure

Technology & processing



Building block: pn junction



Thermal equilibrium

Out of equilibrium

N = carrier density → optical gain

J = current density

τ_s = carrier recombination

d = width of the active zone

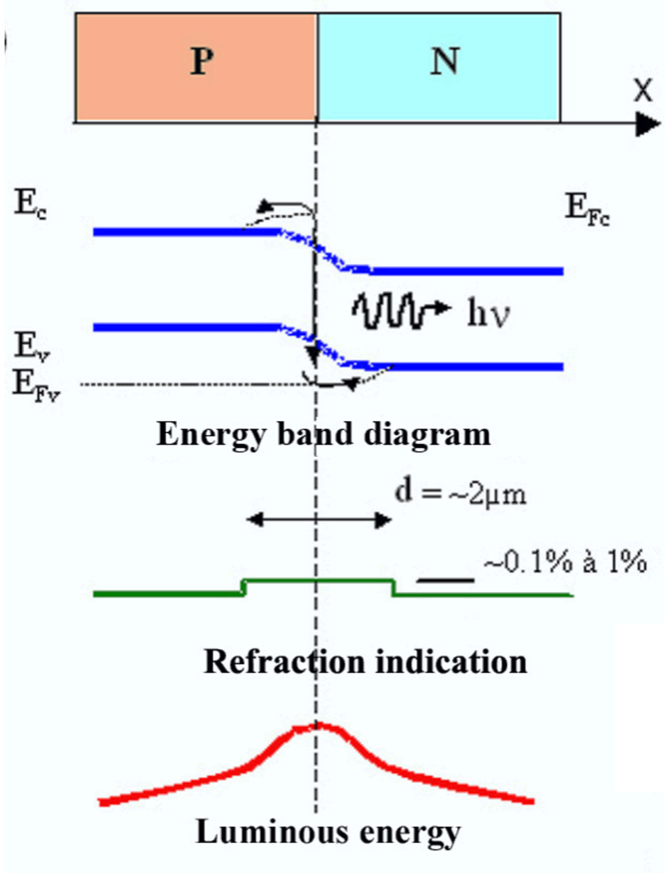
$$N = \frac{J\tau_s}{qd}$$

Homojunction laser

- Homojunction laser

If J ↗ then N ↗
N must be large so that gain and loss balance each other

$$N = \frac{J\tau_s}{qd}$$



In practice

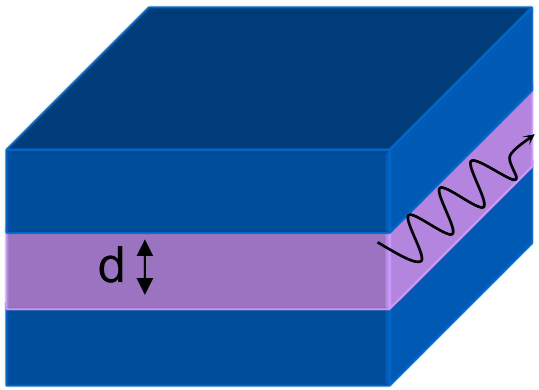
As J is very large ➔ Power dissipation is critical ➔ Strong thermal issues

Only operates at cryogenic temperatures and under pulsed current
→ can not be used in a real communication system

- Double heterostructure

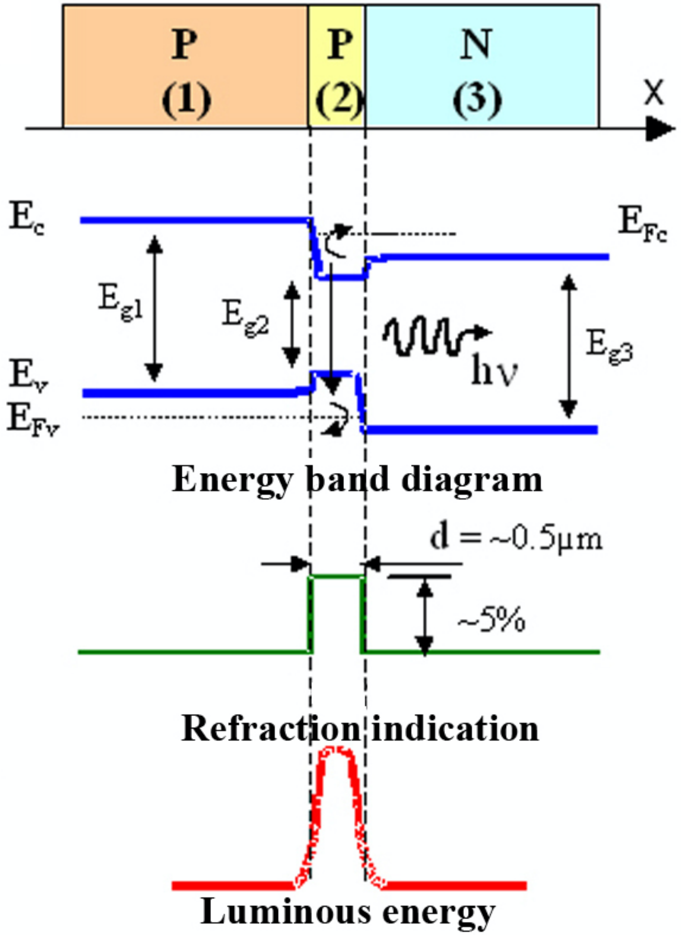
$$N = \frac{J\tau_s}{qd}$$

Gain ↗ means ↘ d



$n_{small\ gap} > n_{wide\ gap}$

Photons are guided !



d < 500 nm against ~ 1 µm for a pn junction
Threshold decreased by one order of magnitude

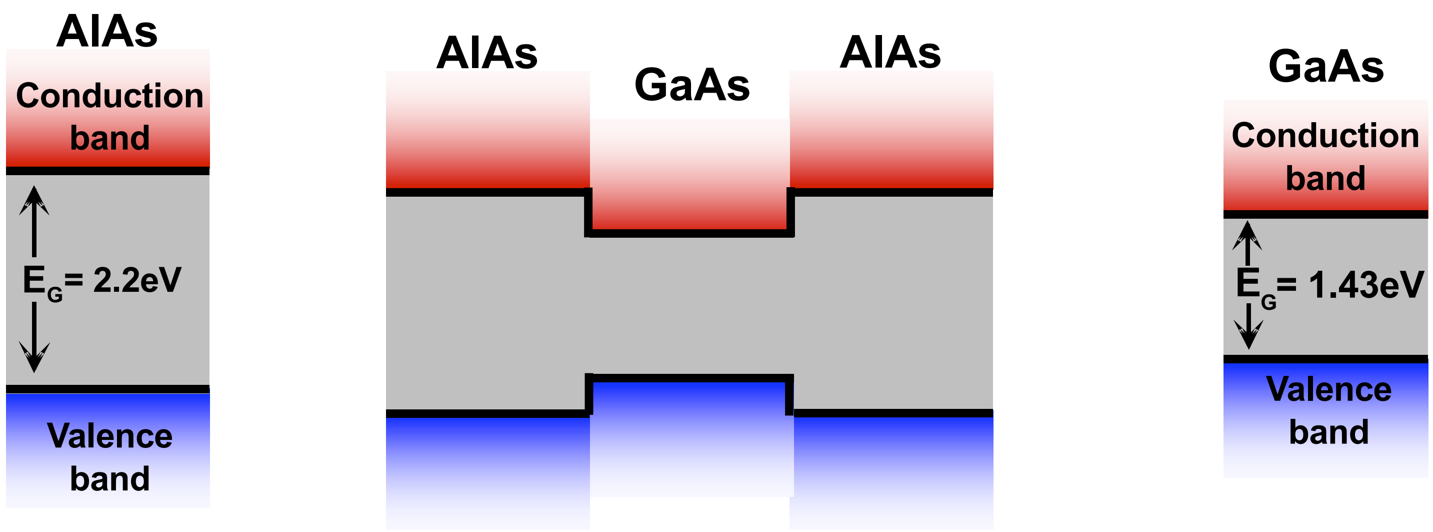
1970: continuous-waves room temperature demonstrated!

Building block: quantum well

Principe: stick two different semiconductor materials

The lattice parameters must be compatible!

Example : GaAs = AlAs = 5.63Å



Quantum well
Small size double heterostructure (nm)

The quantum well is the building block of the quantum technologies

Quantum well technology

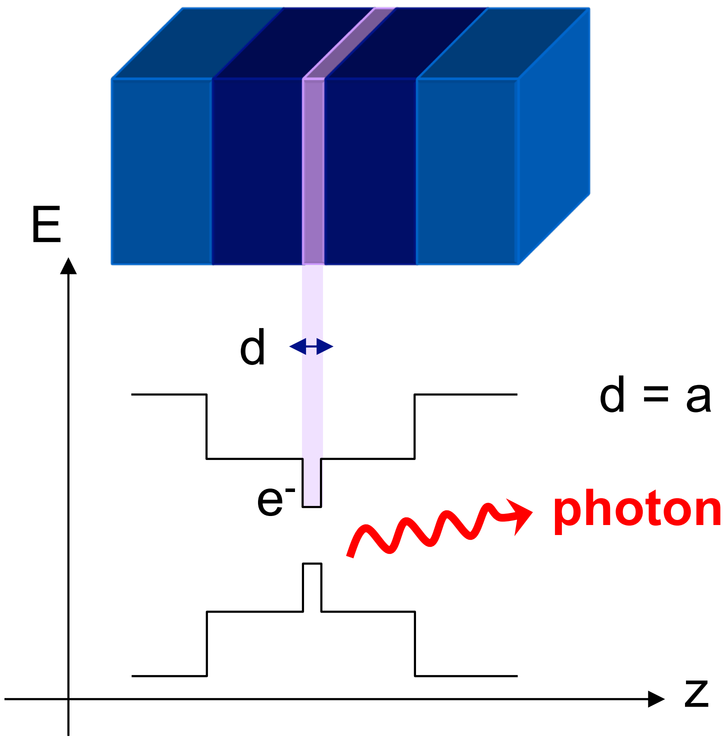
What is the optimal value for "d"?

Gain: the smaller "d", the better!

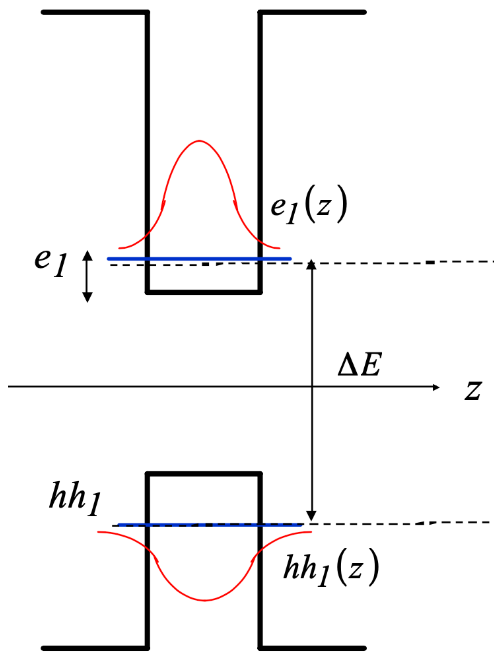
Guiding : If d ~ λ diffraction problem !

Schrödinger's equation for e_1

$$-\frac{\hbar^2}{2m_c}\frac{d^2}{dz^2}e_1(z)+V(z)e_1(z)=e_1.e_1(z)$$



d = a few nm ≈ $\lambda_{\text{De Broglie}}$



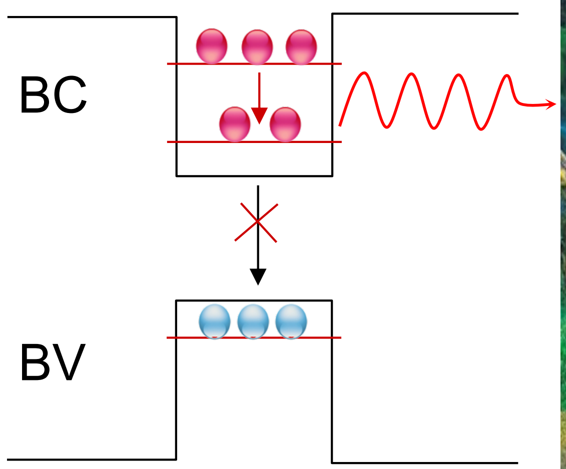
→ diode lasers with enhanced performance e.g., large output power, high-temperature operation, very long lifetime,…

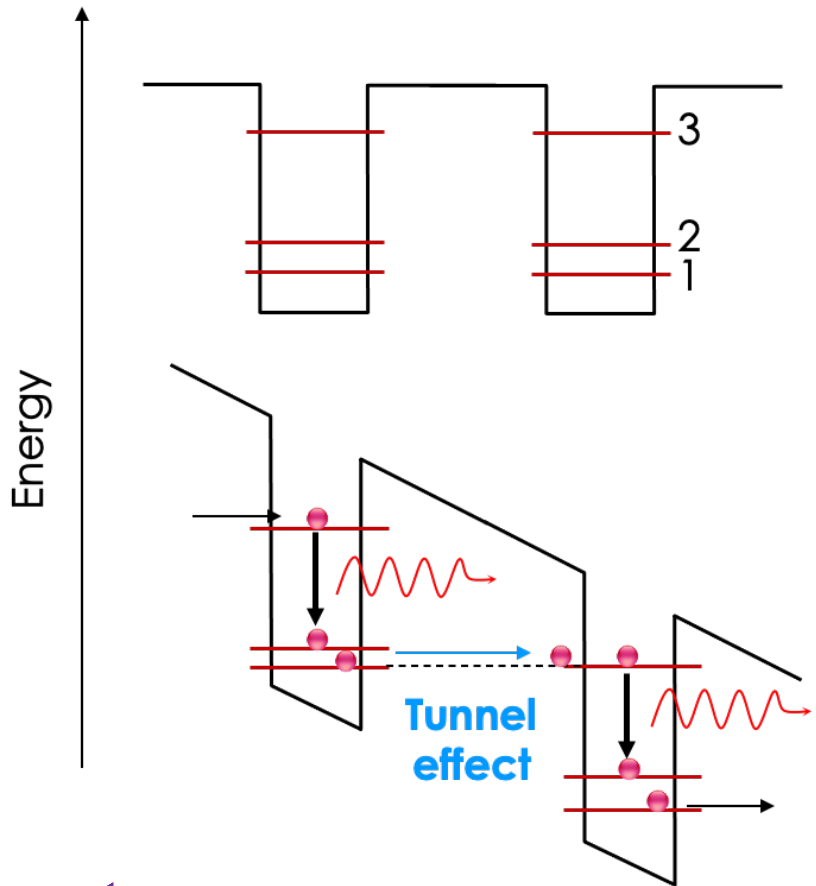
Quantum cascade lasers

Greatly tunable from 3 to 300 microns optical wavelength

Short carrier lifetime (intersubband process with picosecond transitions)

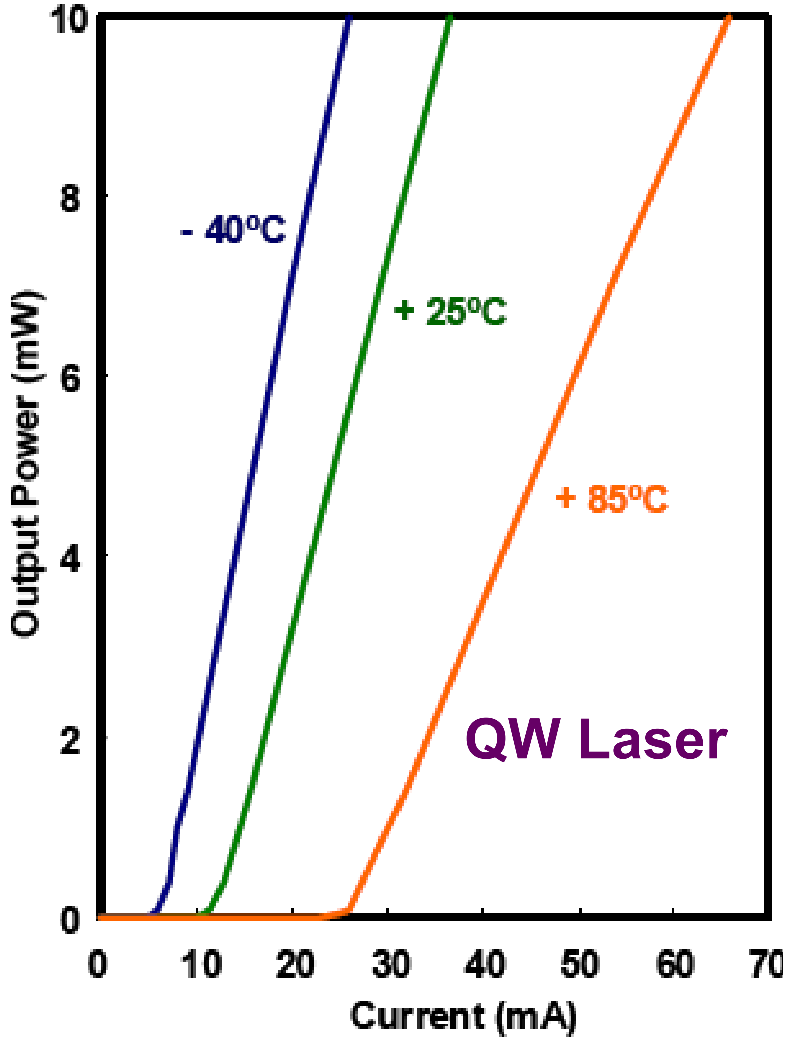
Unipolar: only electrons are involved

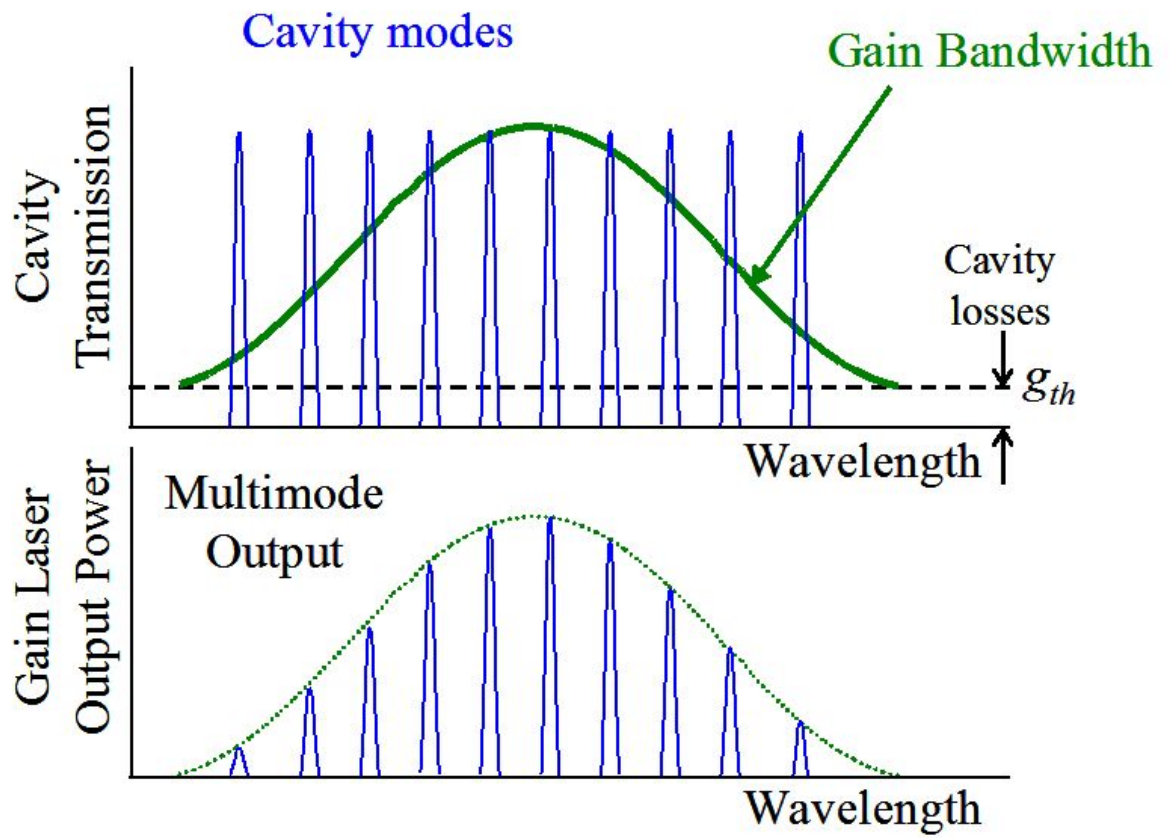




Applications : free-space communications, spectroscopy

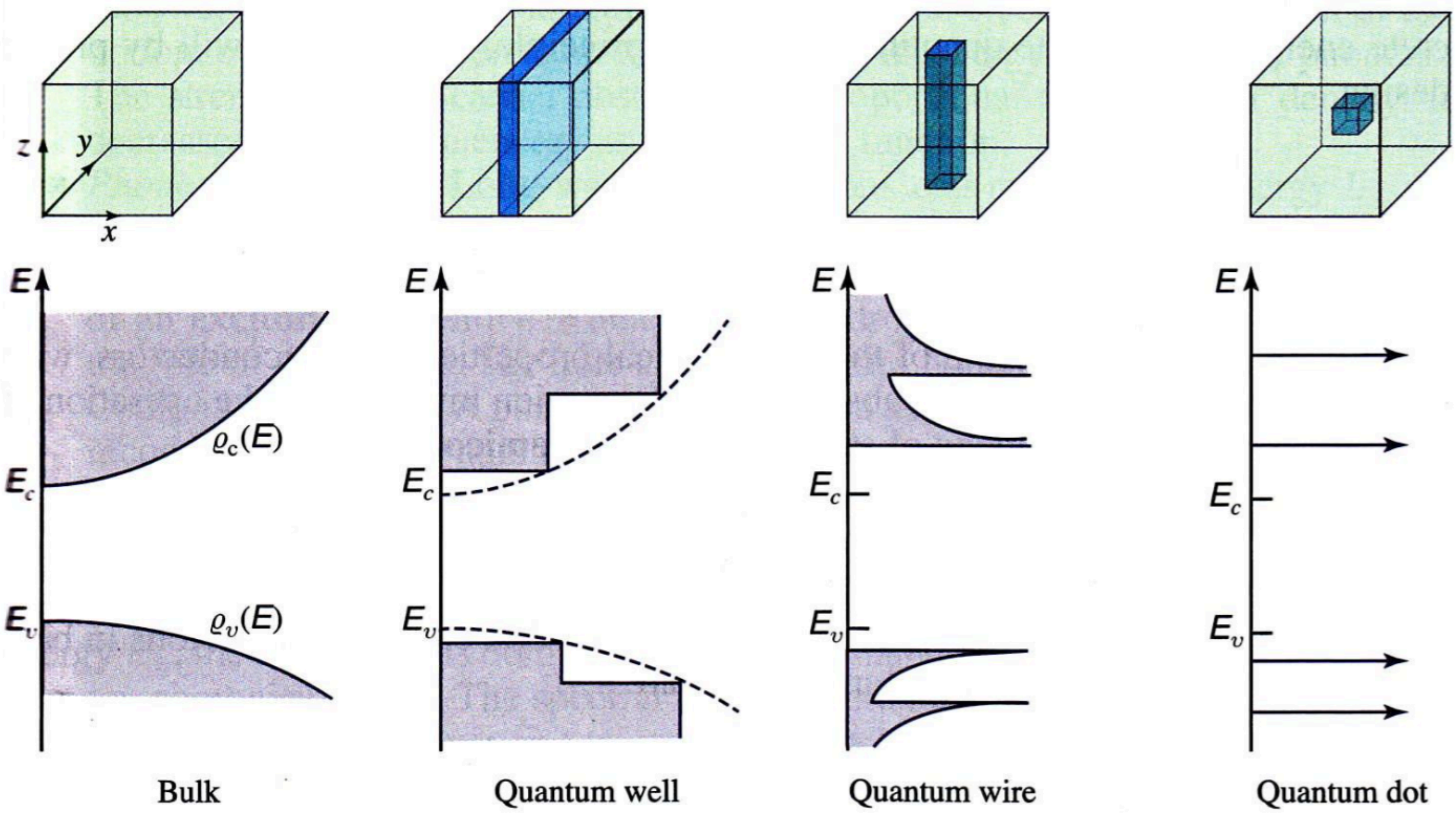
Light current characteristics
Emission properties



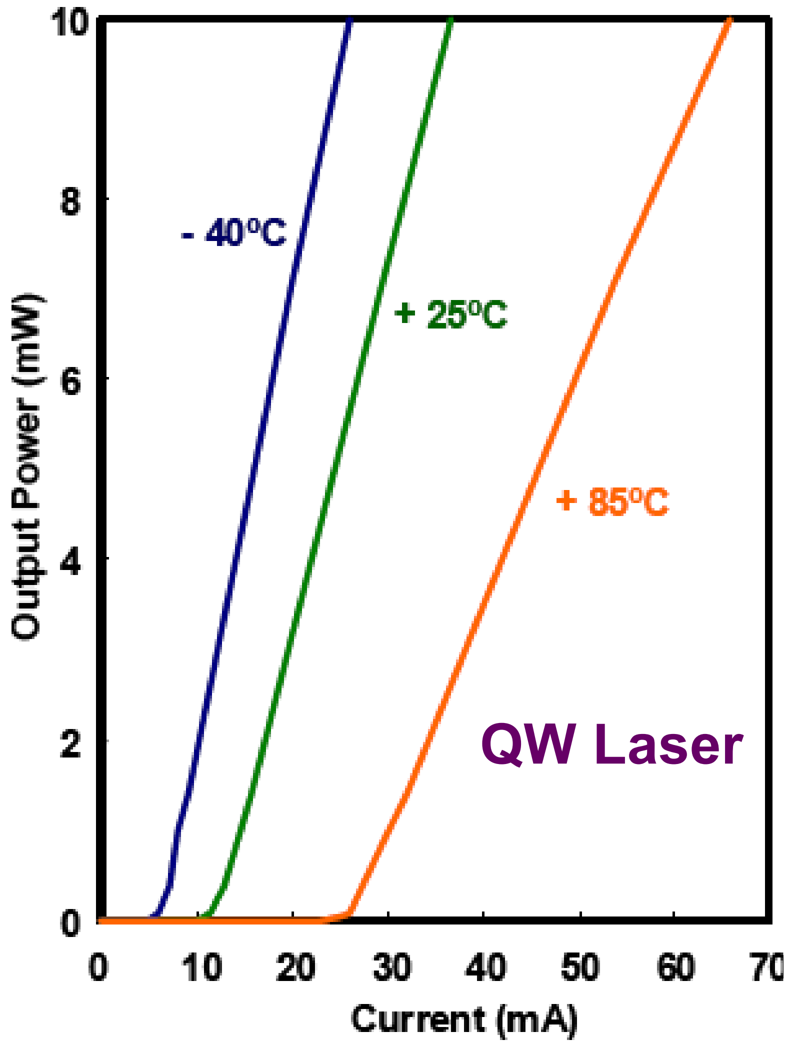


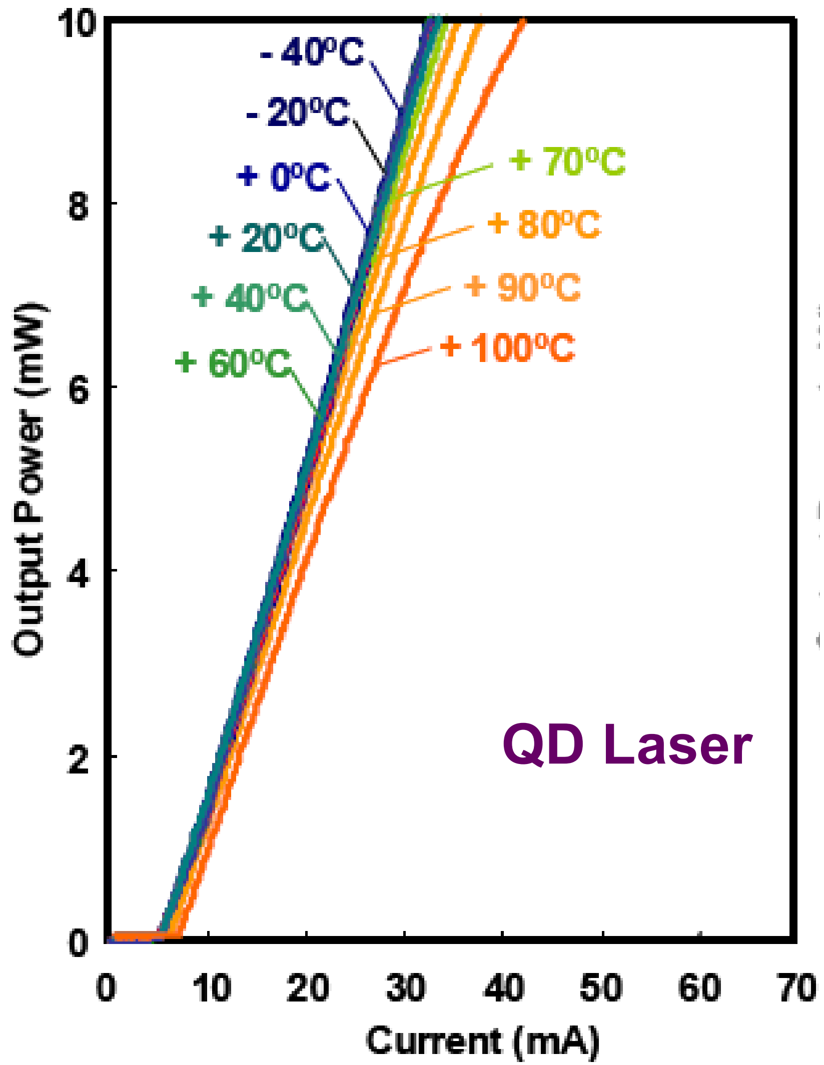
Multimode operation → Fabry-Perot lasers

Low dimensional quantum systems



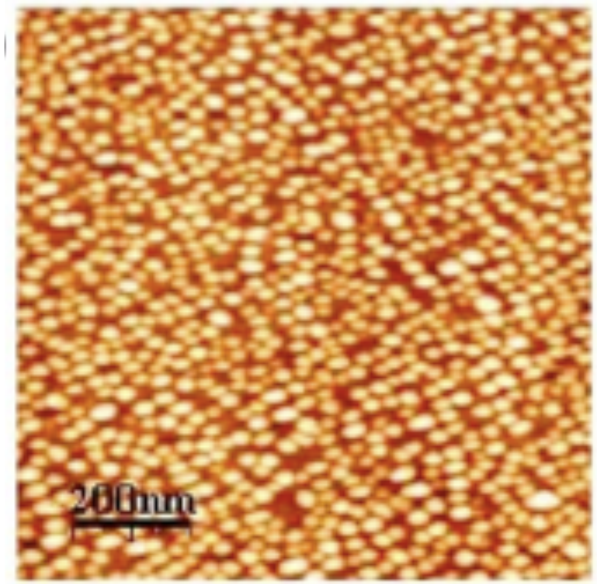
Light current characteristics
Emission properties





Quantum dot lasers

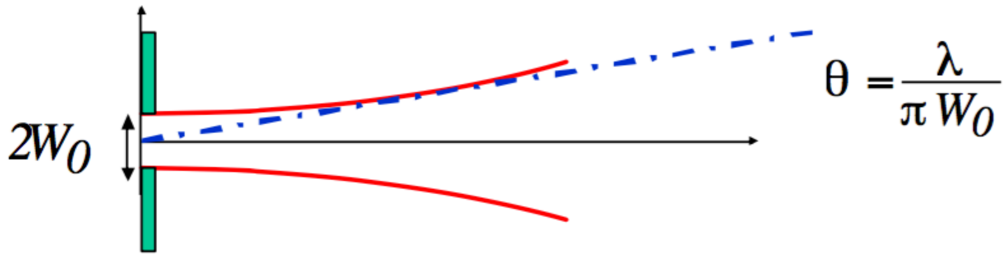
InAs QDs

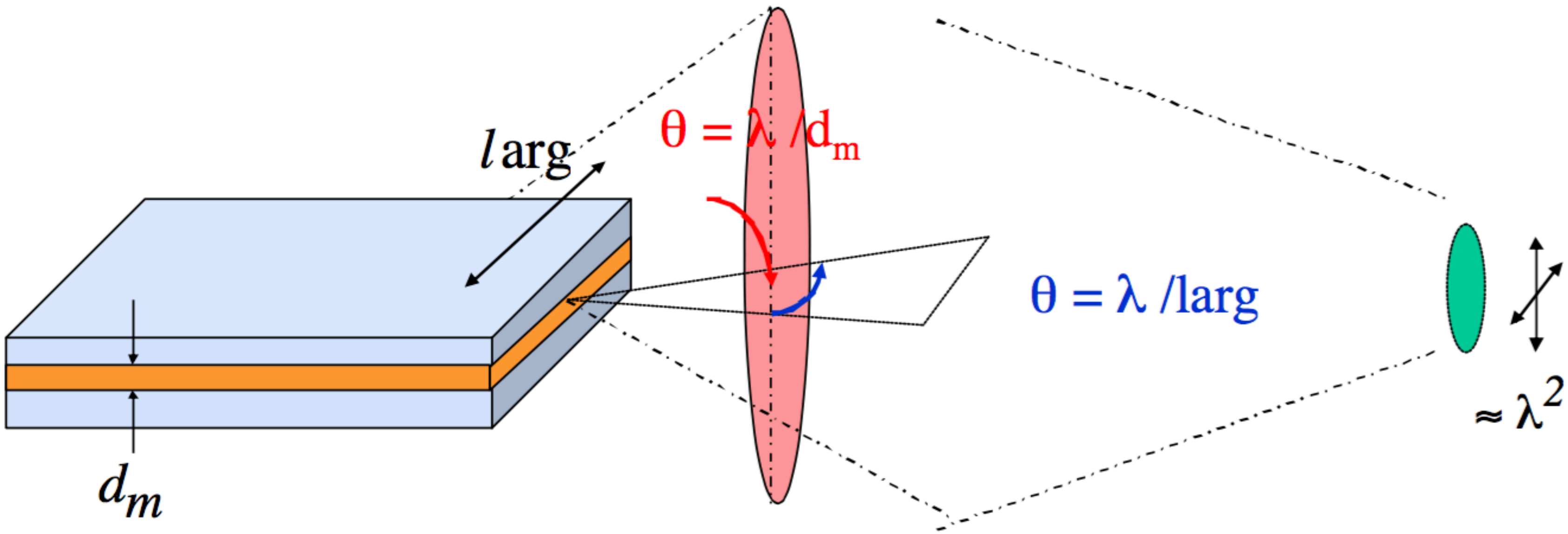


→ Electrons, holes are confined in a tiny structure, so the carriers can hardly move, even at elevated temperatures

Emissivity

Gaussian beam theory





Anisotropic divergence → anamorphosis optics

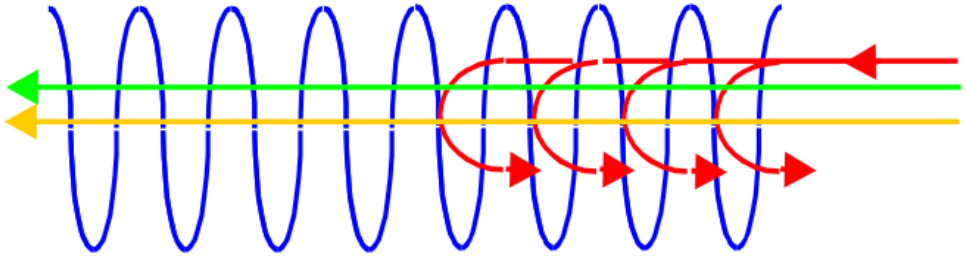
Single-mode laser (DFB)

Periodic perturbations of the refractive index along the laser cavity provide the frequency-selective feedback

DFB lasers are designed to experience internal feedback all along the gain ridge → no end mirrors are needed

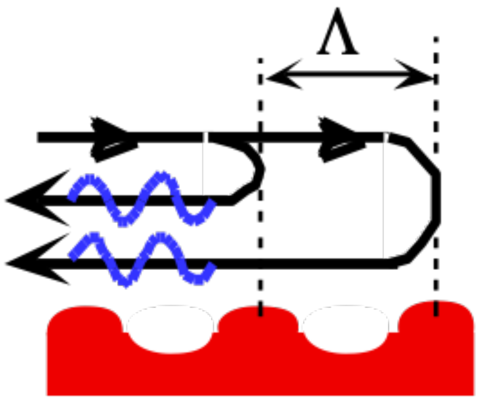
Fabry-Perot
Localized reflection
λ-independent
Many modes

Distributed feedback
Distributed reflection
λ-dependent
Phase π/2
2 or 1 modes

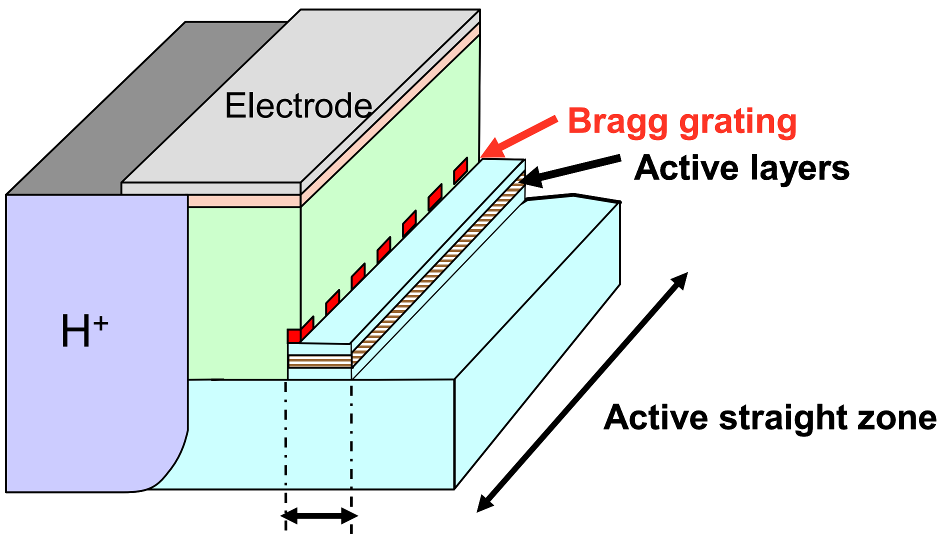


H. Kogelnik and C. V. Shank, J. Appl. Phys. 43 (5), 2327 (1972)

Single-mode laser (DFB)

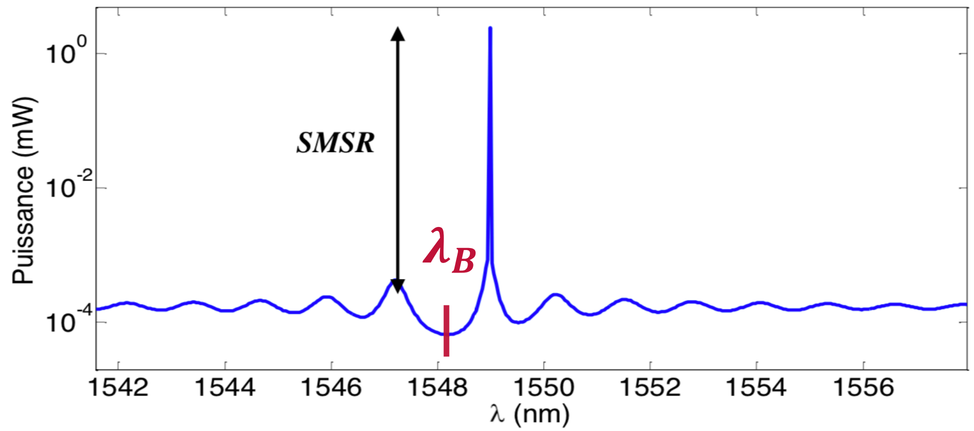


$$q(\lambda_B/2n) = \Lambda$$



Active zone

InAsP QWs	QW Width	Barrier Width	QW Strain	Barrier strain
9	8 nm	8 nm	+1%	-0.6%



Corpuscular equations

$$E(t) = \sqrt{S(t)}\, e^{-j\phi(t)} \qquad S = |E|^2$$

Complex field

Carrier reservoir

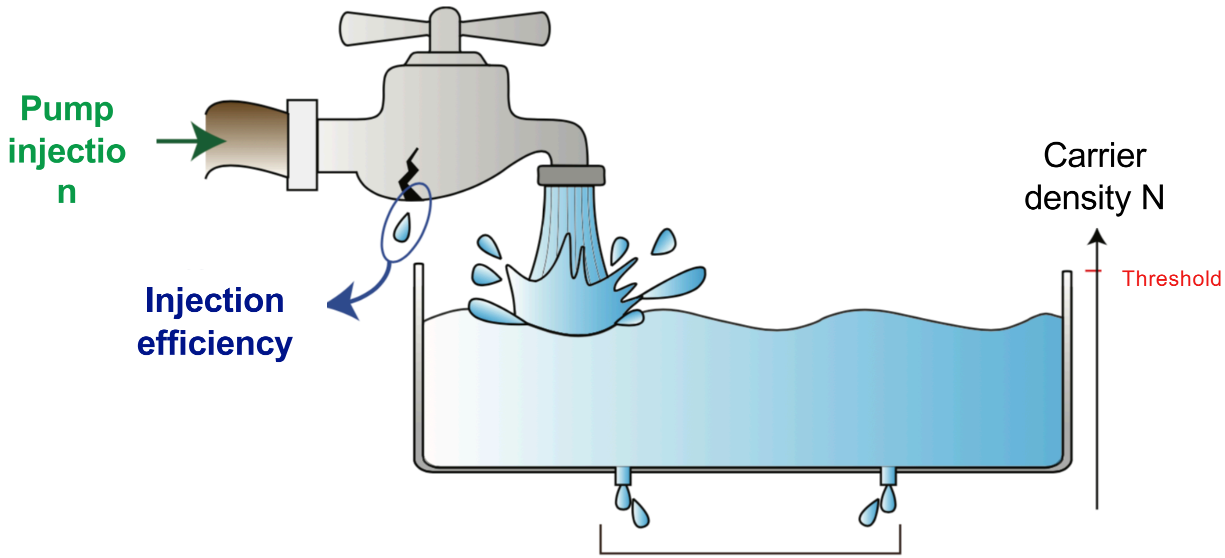
$$\frac{dN}{dt} = \frac{I}{qV} - \frac{N}{\tau_c} - G_N (N - N_t) S$$

$$\frac{dS}{dt} = G_N (N - N_t) S - \frac{S}{\tau_p}$$

Photon reservoir

$$\frac{d\phi}{dt} = \frac{\alpha_H}{2} G_N (N - N_t)$$

Phase



Loss : non-radiative recombination and spontaneous emission

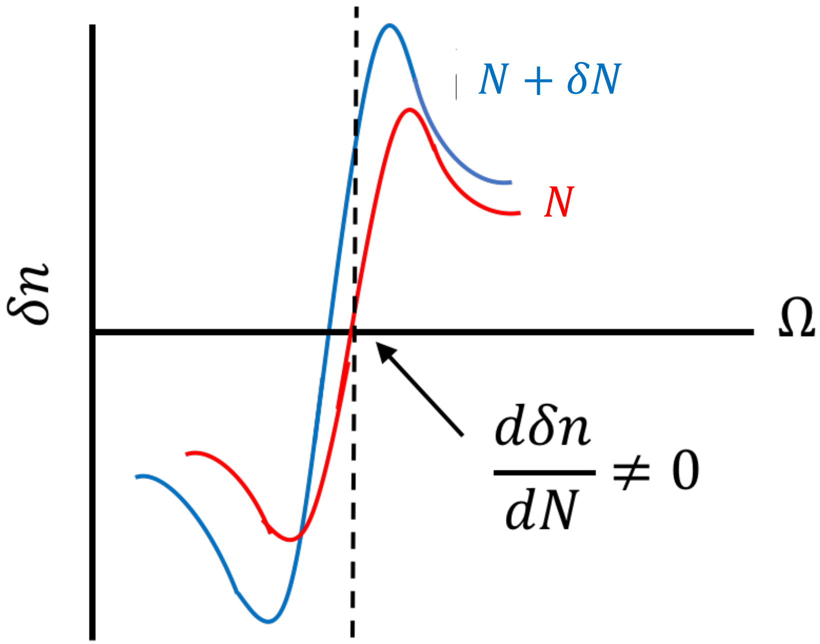
L. A Coldren, S. W Corzine. Diode lasers and photonic integrated circuits, John Wiley & Sons, 2012.

Linewidth broadening factor

Frequency pulling → Change of the refractive index → α_H factor

$$\alpha_H = -\frac{4\pi}{\lambda}\frac{\delta n}{\delta G}$$

Variations of the carrier density modulates the gain and the optical index causing the resonant mode to shift



$$\delta\nu = \frac{\alpha_H}{4\pi} G_N v_g \delta N$$

chirp of the cold cavity

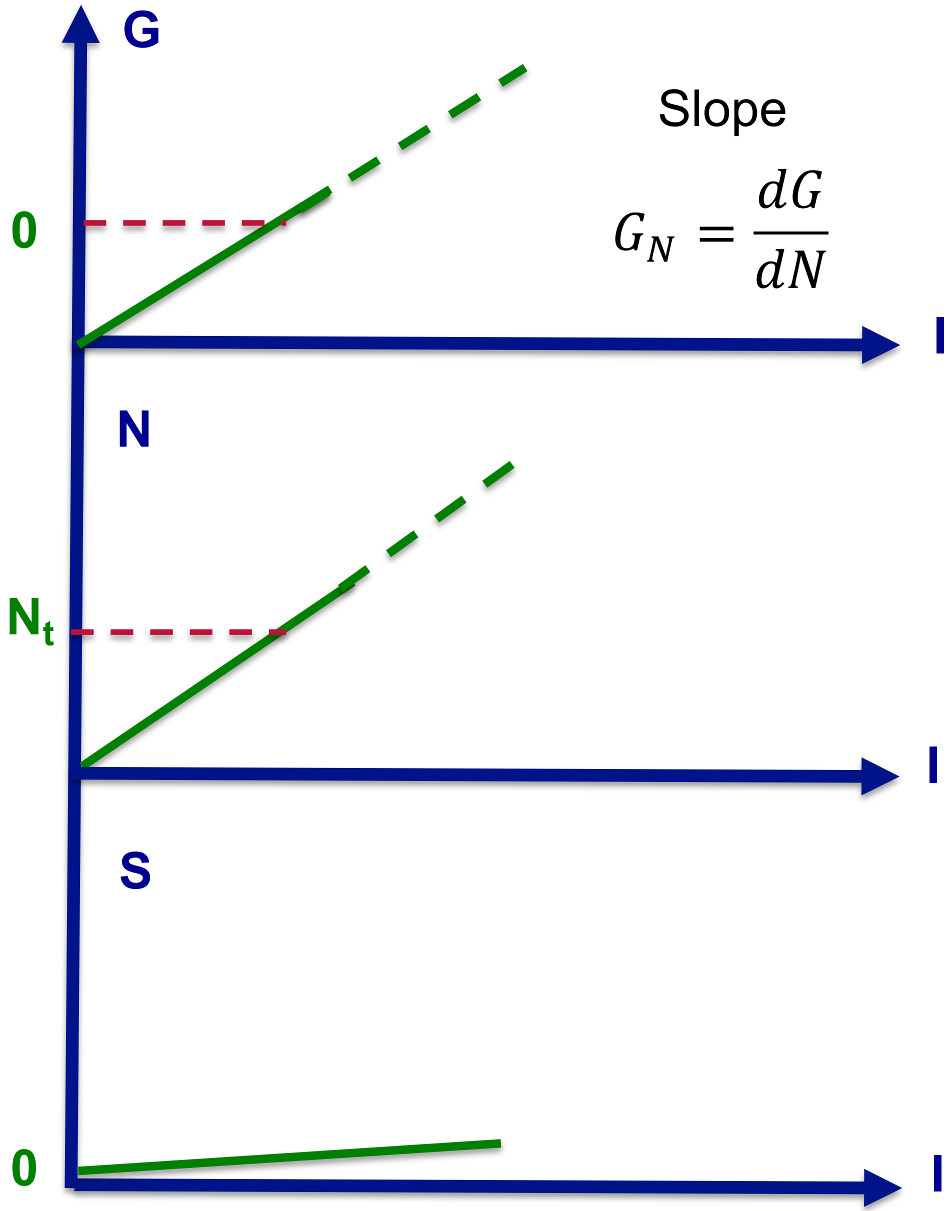
Additional frequency chirping induced by the modulation (see later on)

F. Grillot et al. , IEEE Journal of Quantum Electronics, Vol. 44, pp. 946-951, 2008

Clamping & saturation

$$\frac{dN}{dt} = \frac{I}{qV} - \frac{N}{\tau_c} - G_N\left(N - N_t\right)S$$

$$\frac{dS}{dt} = G_N\left(N - N_t\right)S - \frac{S}{\tau_p}$$



Gain (linear approximation)

$$G(N) = G_N(N - N_t)$$

Cavity frequency

$$\omega(N) = \omega_{th} + \omega_N(N - N_{th})$$

Carrier density at optical transparency

→ $N > N_t$: luminescence (G>0)

→ $N < N_t$: absorption (G<0)

Threshold ($N=N_{th}$): carrier clamping

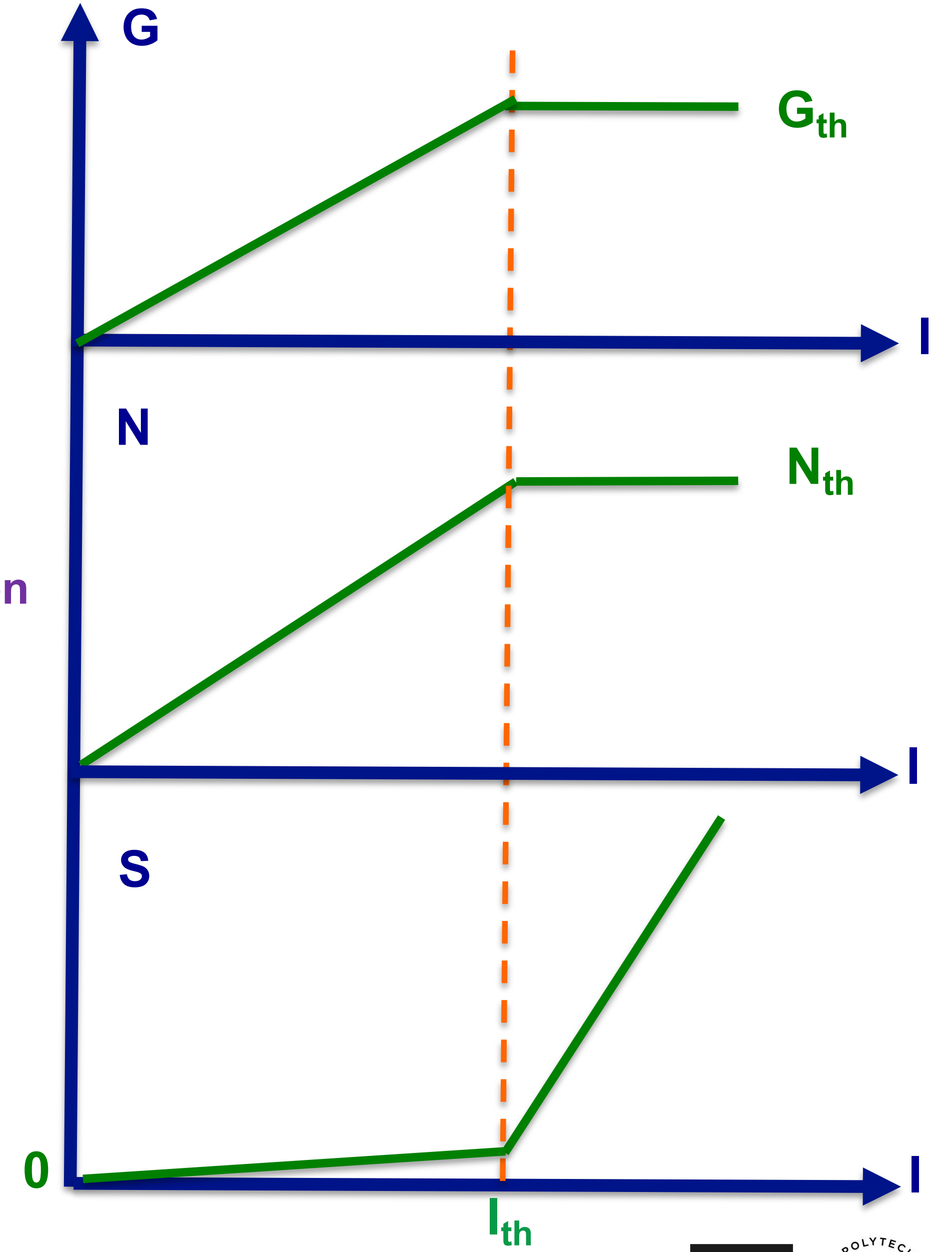
→ Gain = $\sum$ Loss

→ $G(N_{th})=G_{th}$

Clamping & saturation

$$\frac{dN}{dt} = \frac{I}{qV} - \frac{N}{\tau_c} - G_N\left(N - N_t\right)S$$

$$\frac{dS}{dt} = G_N\left(N - N_t\right)S - \frac{S}{\tau_p}$$



Gain saturation

→ **Stabilization when stimulated recombination are compensated by the pump**

$$N = \frac{N_0}{1 + \frac{S}{S_{sat}}} \quad \Rightarrow \quad \boxed{G = \frac{G_0}{1 + \frac{S}{S_{sat}}}}$$

$$N_0 = \frac{I\tau_c}{eV} \qquad G_0 = G_N N_0 \qquad Ssat = \frac{1}{G_N \tau_c}$$

Relaxation oscillations

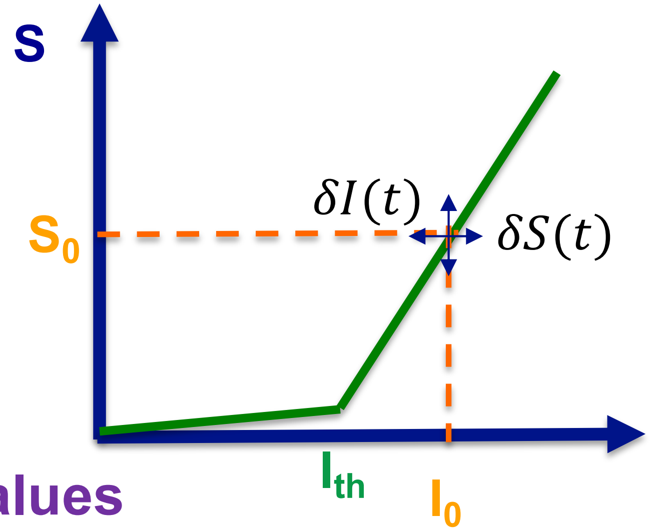
$$\frac{dN}{dt} = \frac{I}{qV} - \frac{N}{\tau_c} - G_N\left(N - N_t\right)S$$

$$\frac{dS}{dt} = G_N\left(N - N_t\right)S - \frac{S}{\tau_p}$$

Small perturbation of the steady-state

$$N(t) = N_0 + \delta N(t)$$
$$I(t) = I_o + \delta I(t)$$
$$S(t) = S_0 + \delta S(t)$$

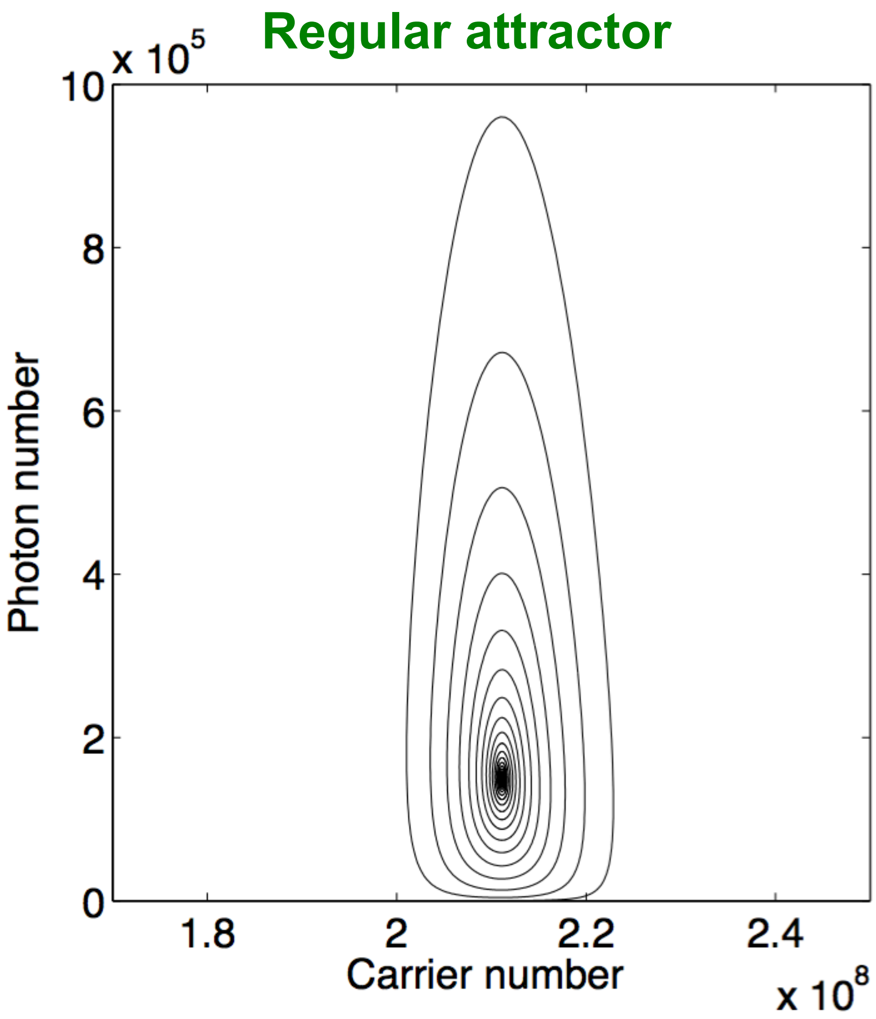
$$\begin{bmatrix} \delta\dot{N} \\ \delta\dot{S} \end{bmatrix} = \begin{bmatrix} -\frac{1}{\tau_c} - G_N S_0 & -1/\tau_p \\ G_N S_0 & 0 \end{bmatrix} \begin{bmatrix} \delta N \\ \delta S \end{bmatrix}$$

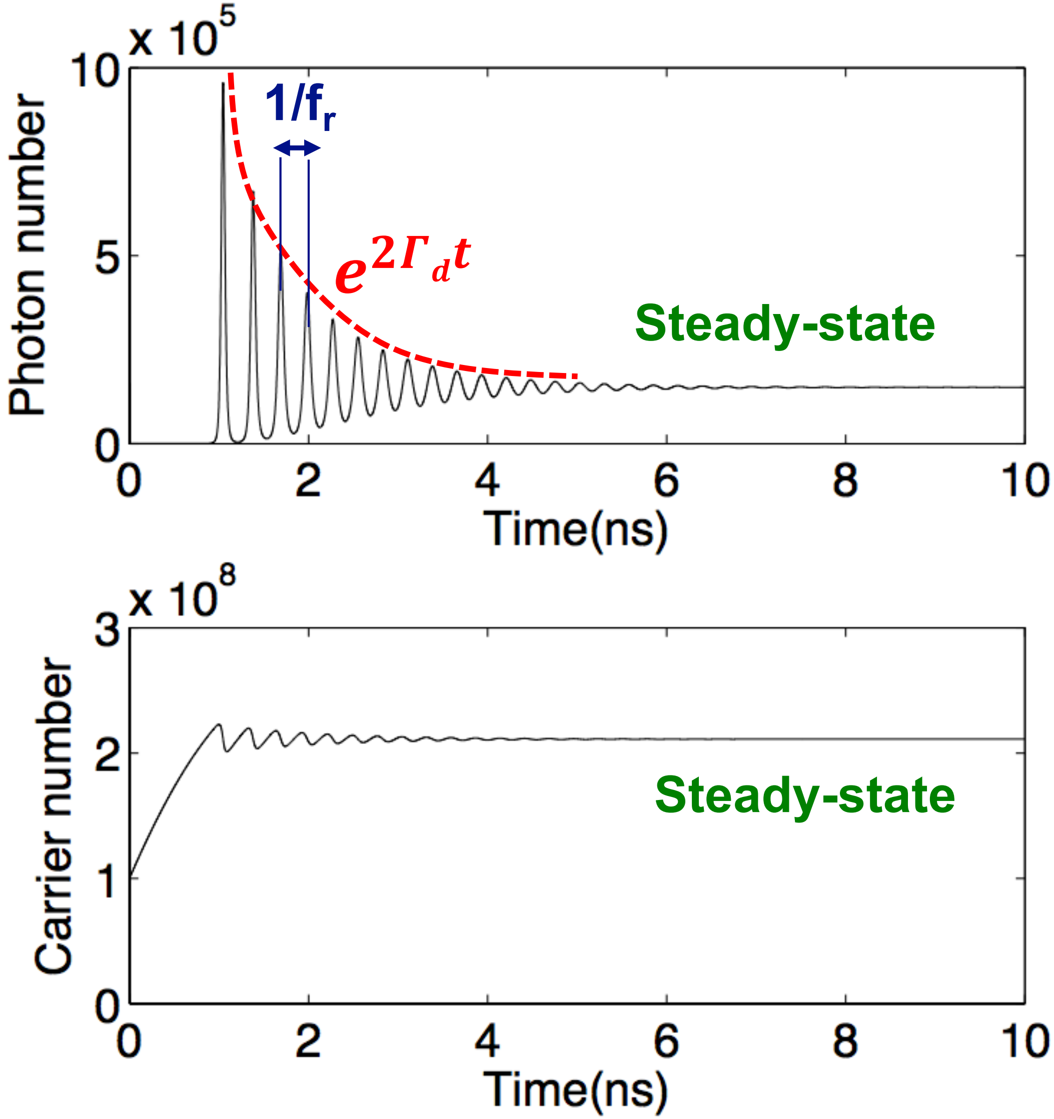


The evolution matrix gives access to the eigenvalues

$$f_{\pm} = -\Gamma_d \pm \sqrt{\Gamma_d^2 - f_r^2}$$ with $\Gamma_d < f_r$ for an interband diode laser

Relaxation oscillations





T. Erneux and P. Glorieux, Laser Dynamics, Cambridge

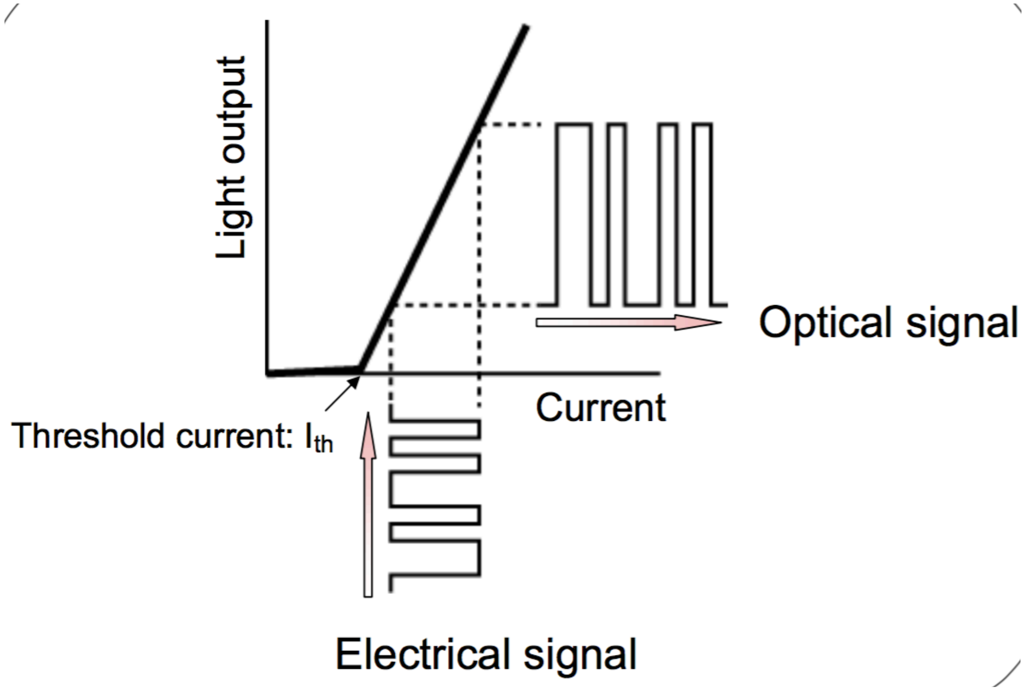
Relaxation oscillations

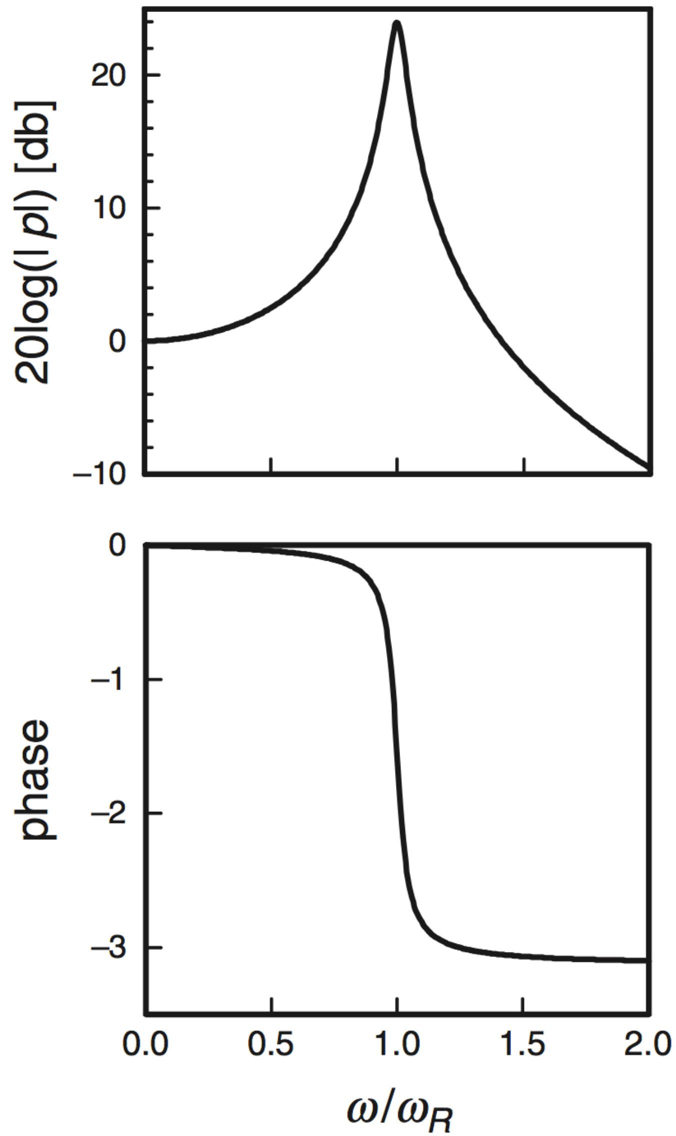
Relaxation oscillation: beating between carrier and photon populations

Semiconductor lasers display very large relaxation frequencies (>GHz) allowing very high-speed modulation

	Population lifetime (carrier lifetime) [s]	Photon lifetime [s]	Relaxation oscillation frequency [Hz]
Semiconductor lasers	10^{-9}	10^{-12}	$\sim 10^{9}$
Solid-state lasers	10^{-3}	10^{-9}	$\sim 10^{5}$
Gas lasers	10^{-8}	10^{-7}	$\sim 10^{6}$

Direct modulation





Laser's modulation response

$$H(j\omega) = \frac{\delta S}{\delta I} = \frac{1}{1 - \omega^2/\omega_r^2 + 2j\omega\Gamma_d/\omega_r^2}$$

$$\phi(j\omega) = \frac{\delta \phi}{\delta I} = \arctan\left(-\frac{2\omega\Gamma_d}{\omega_r^2 - \omega^2}\right)$$

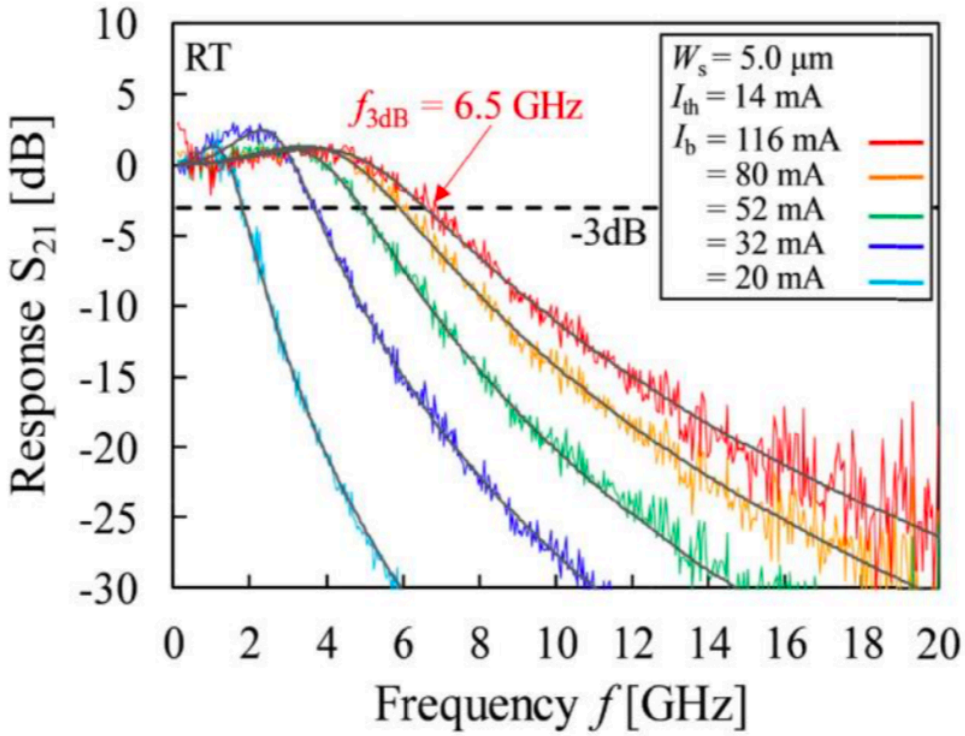
Direct modulation

The modulation response peaks at the relaxation oscillation frequency

$$f_r^2 = \frac{1}{4\pi^2}\frac{G_N S_0}{\tau_p}$$

The 3dB modulation bandwidth scales with the oscillation frequency

$$f_{3dB} = f_r\sqrt{1+\sqrt{2}}$$



At high bias & high modulation frequencies, the modulation response falls down (damping effect, carrier transport across the junction)

Modulation dynamics

How to increase the modulation speed ? Need to enhance the relaxation oscillation frequency & the 3dB modulation bandwidth

→ **Increasing the photon density via a better confinement of the optical field in the active layer**

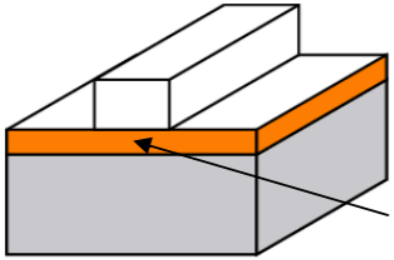
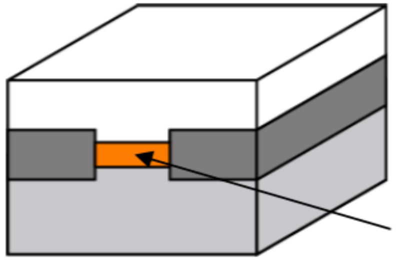
→ **Biasing the laser at a higher pump current**

→ **Increasing the differential gain G_N coefficient by cooling the device, doping active areas or by using quantum confined semiconductors (wells, dots)**

→ **Reducing the photon lifetime by decreasing the laser cavity length**

Modulation dynamics

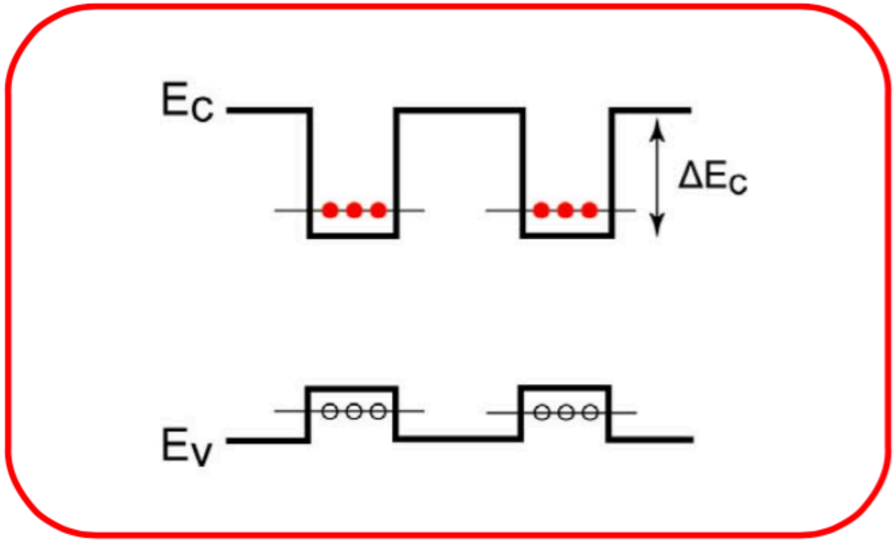
Decreasing active-region volume e.g. depends on waveguide structure

	Ridge waveguide structure 	Buried heterostructure (BH) 
Active region	Defined by ridge width and current spreading	Defined by mesa width
Width	Relatively wide (Usually > 2 μm)	Narrow (Usually < 1.5 μm)
Fabrication	Etching of ridge	Etching of mesa and regrowth
Point	Control of guided optical mode and current spreading	Suppression of leakage current at regrowth interface and current blocking structure

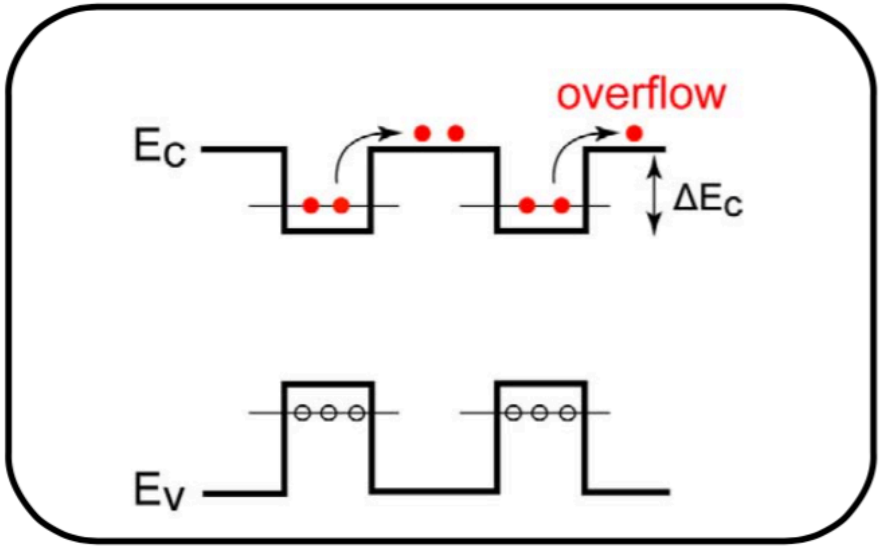
Modulation dynamics

Increasing differential gain G_N → quantum well semiconductors

AlGaInAs quantum well (QW)



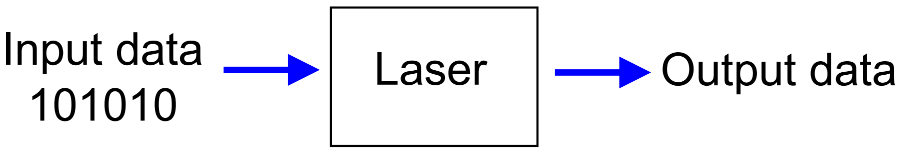
InGaAsP quantum well (QW)



Superior band diagram of AlGaInAs QW
→ Large conduction band offset ΔE_c
→ Small valence band offset ΔE_v

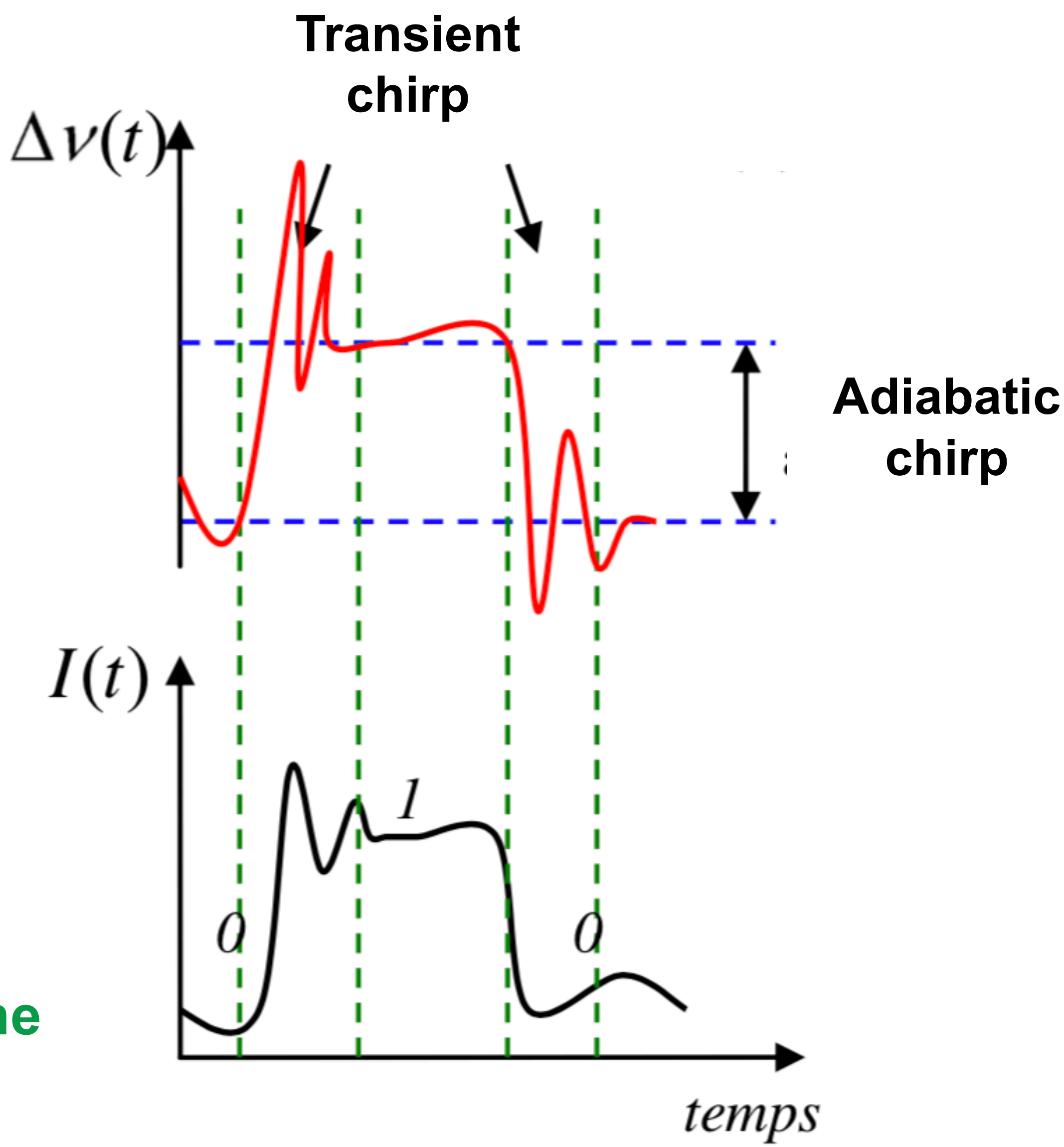
Better electron confinement → Increase modulation efficiency

Frequency chirping (dynamic)



Transient Adiabatic

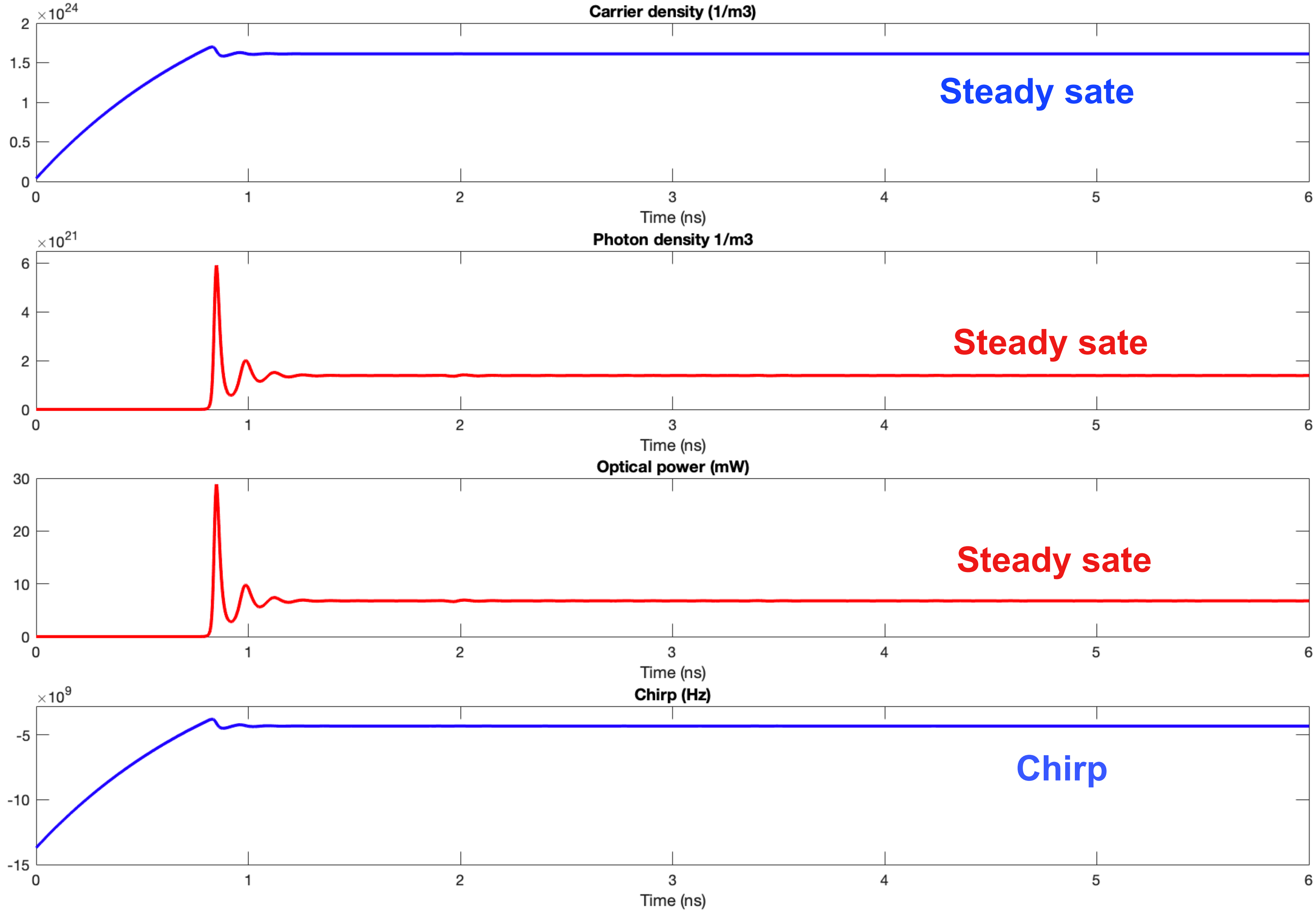
$$\Delta\upsilon = \frac{\alpha_H}{4\pi}\left(\frac{d}{dt}\ln S(t) + \kappa S(t)\right)$$



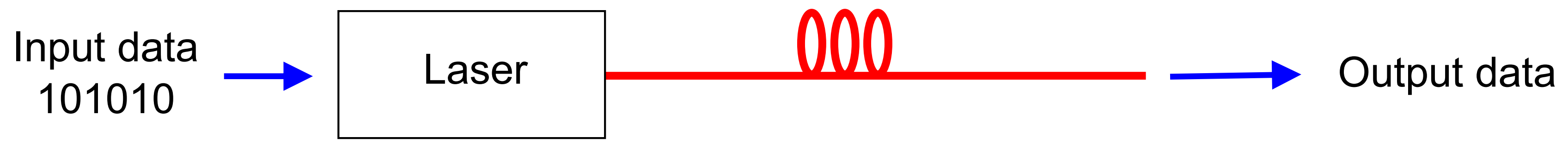
$\alpha_H = 0$ → chirpfree lasers

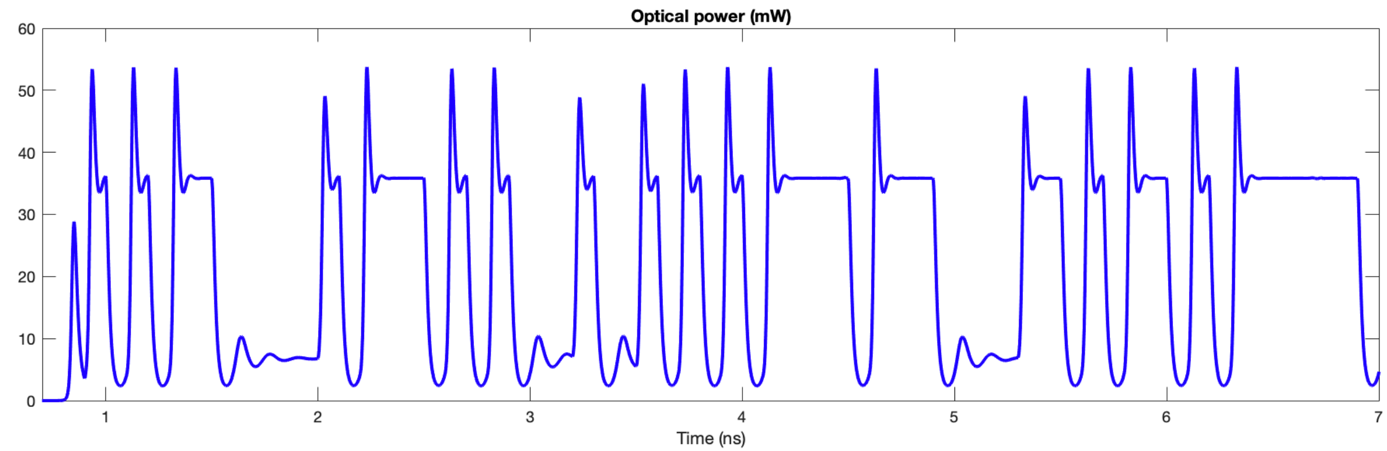
No signal degradation during the transmission

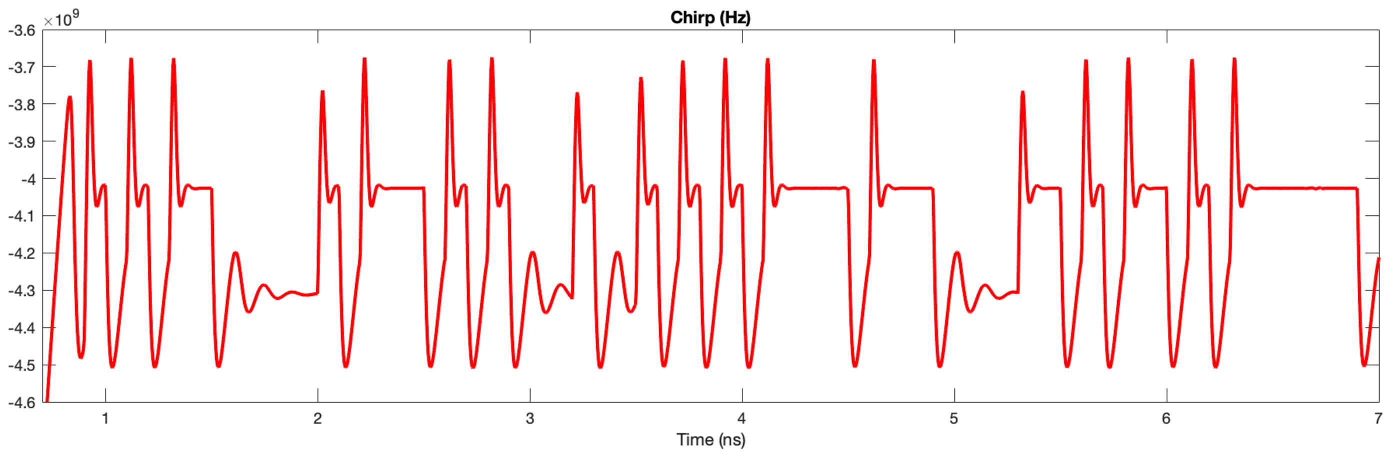
Frequency chirping (static)



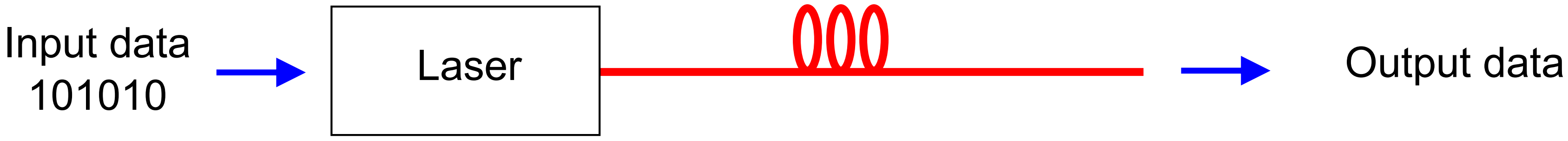
Frequency chirping (dynamic)

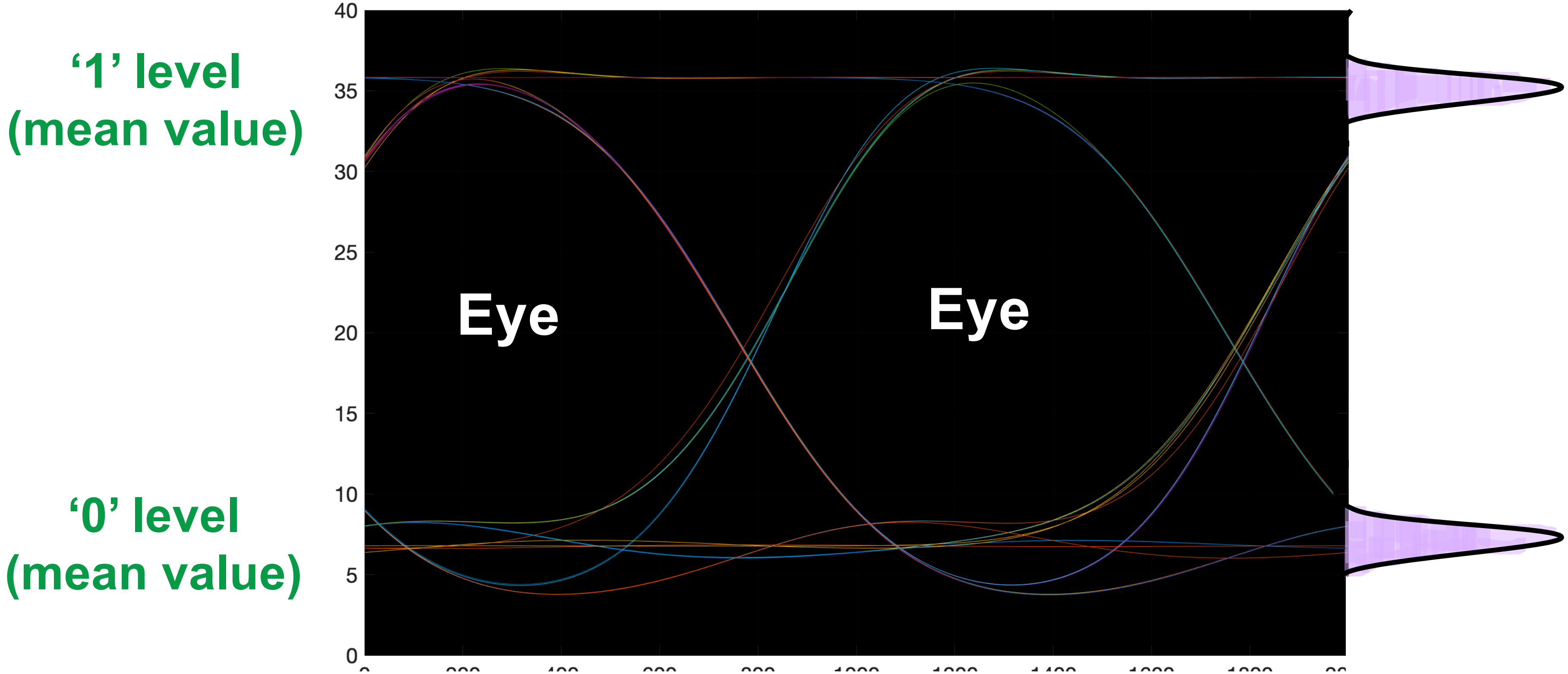






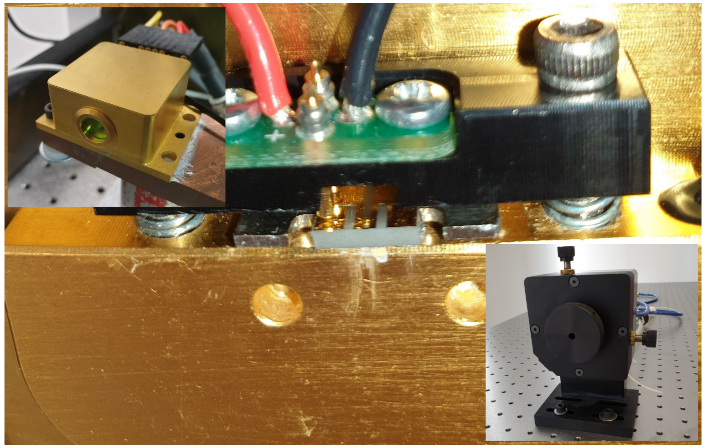
Eye diagram

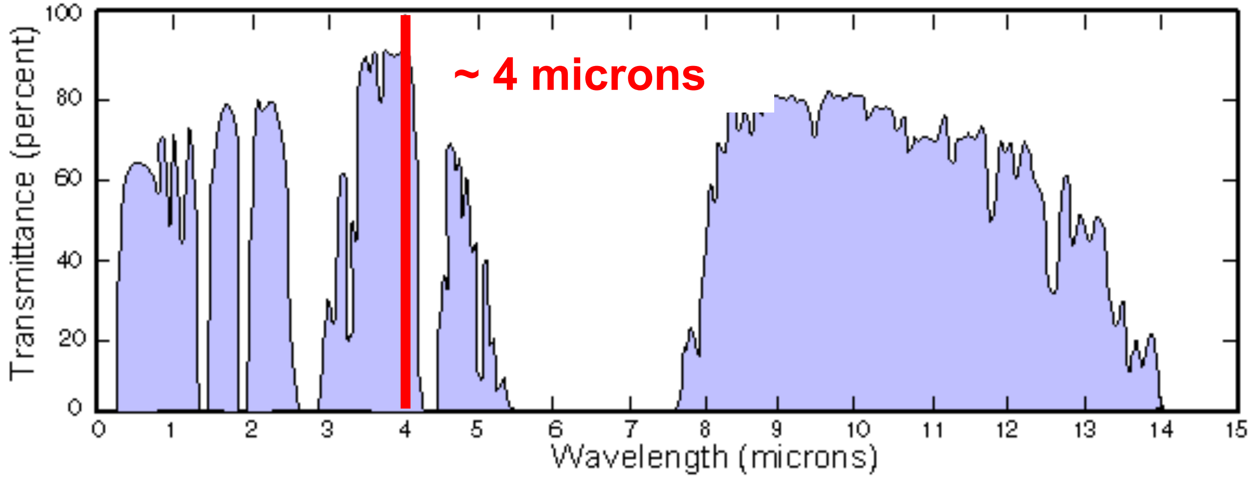




The eye diagram shows the effects of noise, jitter, distortion, inter-symbol interference, and crosstalk, all of which can close the eye and alter the data transmission

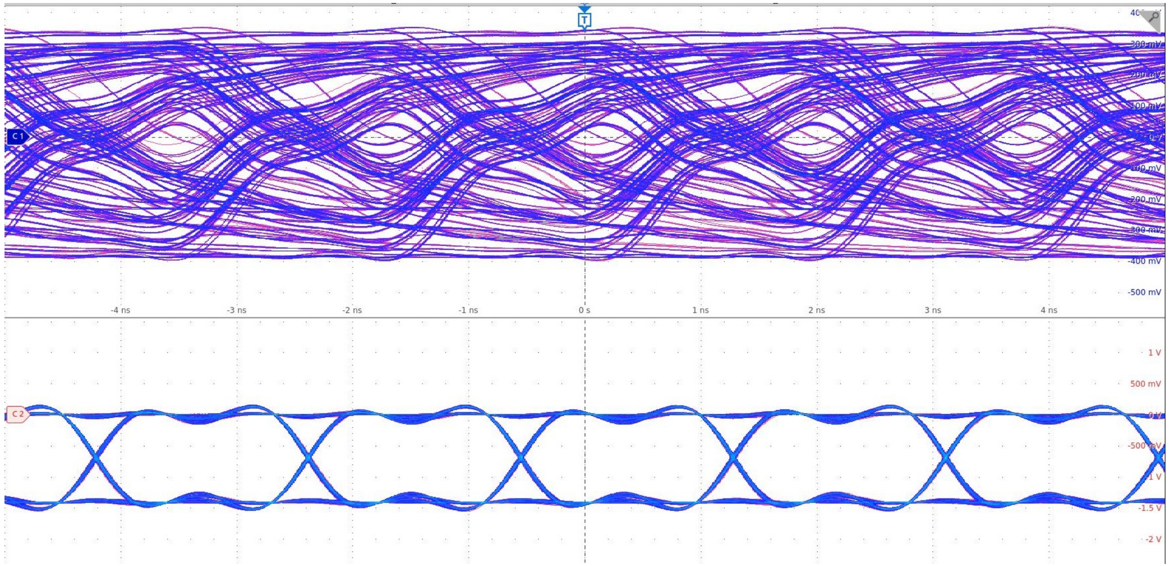
Data transfer with quantum cascade lasers





Detector signal

Seed signal



550 Mb/s

Nonlinear dynamics of semiconductor lasers

Outline

- Introduction to chaos theory
- Lorenz-Haken model
- Laser classifications
- Private communications with chaos
- Random bit generation
- Chaos for light detection & ranging

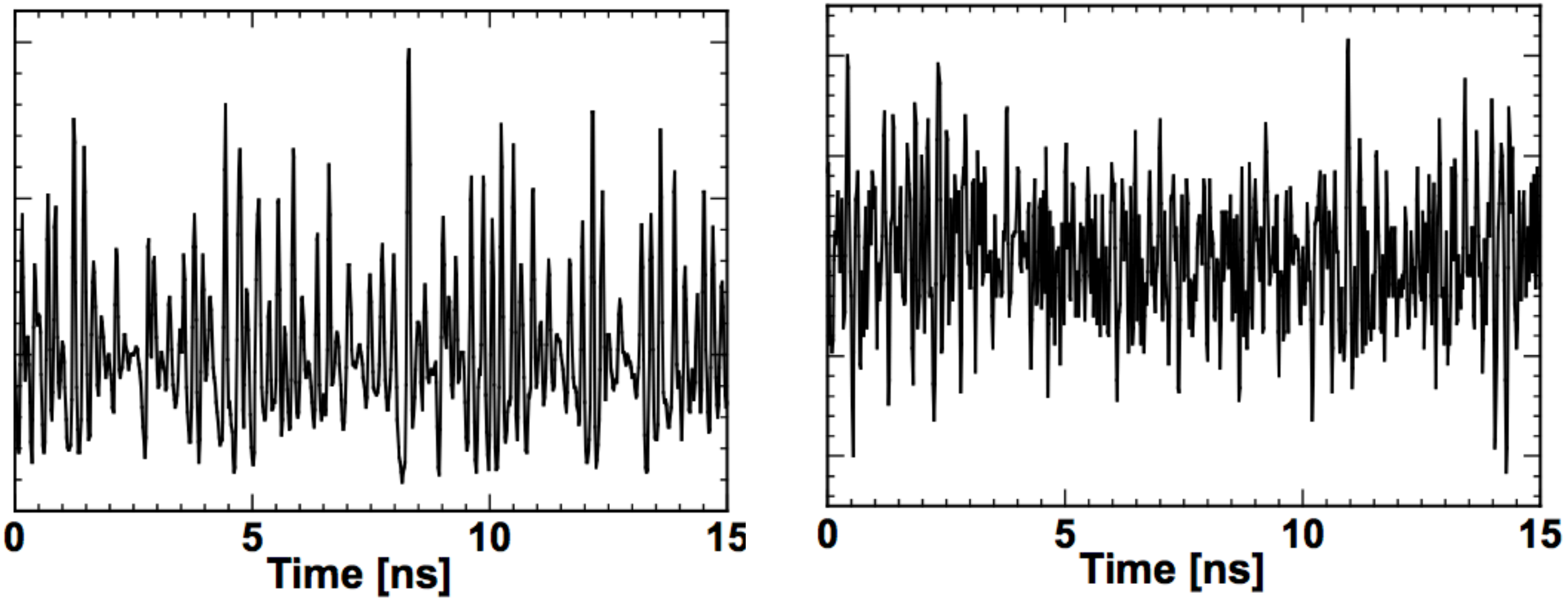
The three revolutions in Physics

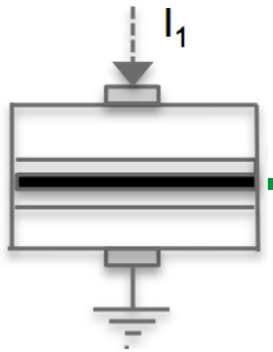
- Relativity theory
- Quantum mechanics
- Chaos theory

Chaos is not an invention but a real discovery!

Chaos versus noise?

Temporal waveforms of laser intensity output in the diode laser

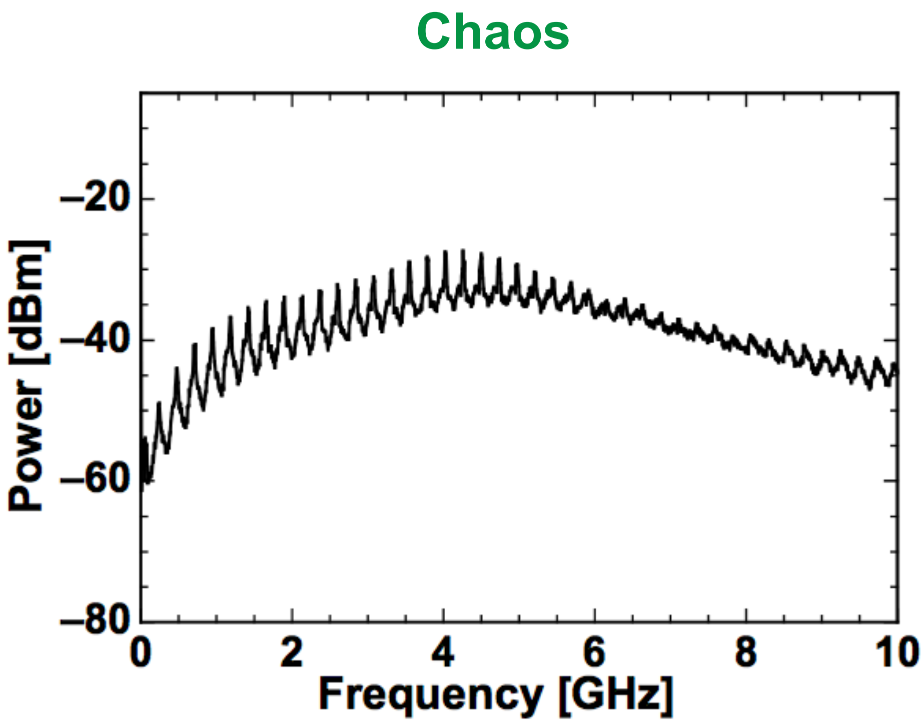


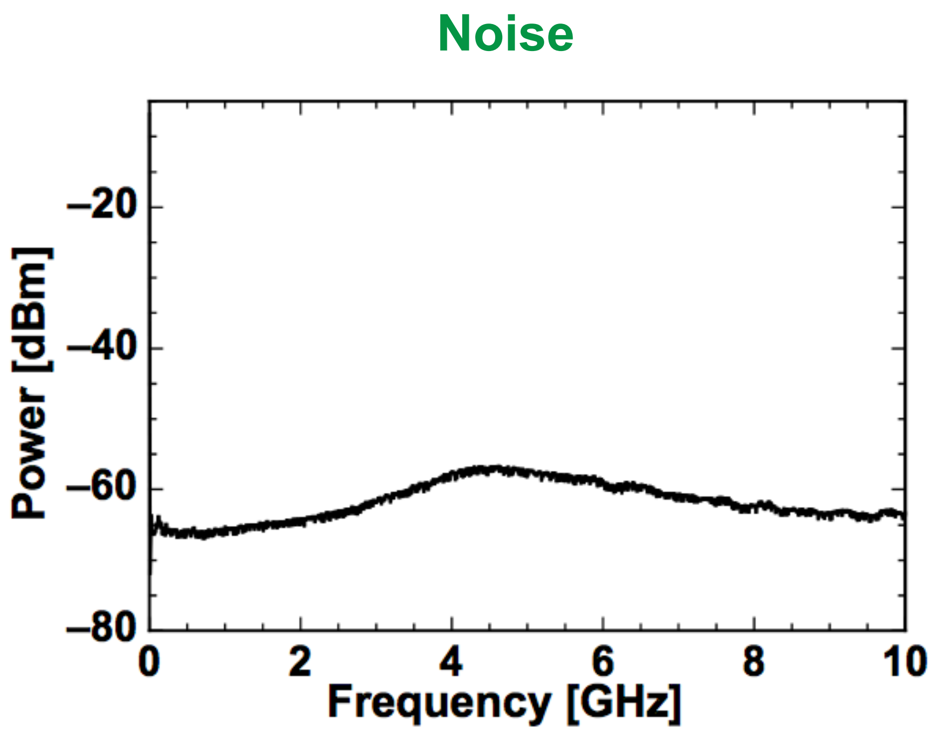


Optical power P(t)

Chaos versus noise?

Temporal waveforms of laser intensity output in the diode laser





Chaos is fundamentally driven by a characteristic frequency of the nonlinear system e.g. for a diode laser, the relaxation frequency (see later on)

Navier-stokes equations

- Weather prediction

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial (\rho u^2)}{\partial x} + \frac{\partial (\rho u v)}{\partial y} + \frac{\partial (\rho u w)}{\partial z} = -\frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \tag{2}$$

$$\frac{\partial \rho v}{\partial t} + \frac{\partial (\rho u v)}{\partial x} + \frac{\partial (\rho v^2)}{\partial y} + \frac{\partial (\rho v w)}{\partial z} = -\frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \tag{3}$$

$$\frac{\partial \rho w}{\partial t} + \frac{\partial (\rho u w)}{\partial x} + \frac{\partial (\rho v w)}{\partial y} + \frac{\partial (\rho w^2)}{\partial z} = -\frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \tag{4}$$

$$\frac{\partial \rho E}{\partial t} + \frac{\partial (\rho u E)}{\partial x} + \frac{\partial (\rho v E)}{\partial y} + \frac{\partial (\rho w E)}{\partial z} = -\frac{\partial p u}{\partial x} - \frac{\partial p v}{\partial y} - \frac{\partial p w}{\partial z} + S \tag{5}$$

where ρ is the air density, u, v, w are the components of the air's velocity, E is measure of the air's internal energy (which allows us to compute its temperature) and p is the air pressure.

These equations can not be solved analytically and even numerically it requires the use of a super computer!

Brian J. Cantwell, Fundamentals of Compressible Flows, Course from Stanford University

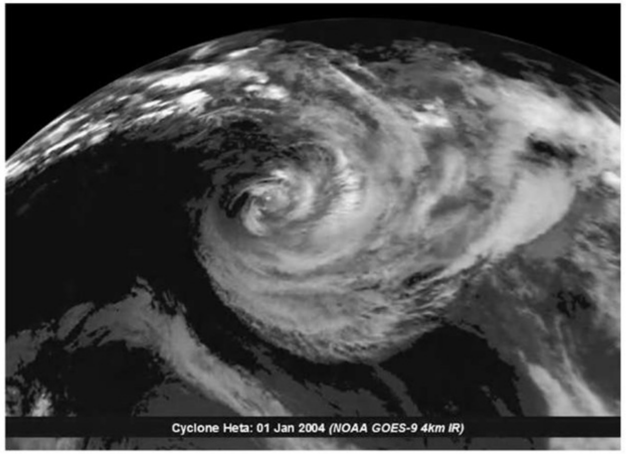
The Lorenz model

- Edward Lorenz gave rise to the modern field of chaos theory. His model consists on a very simplified approach made with three coupled ordinary differential equations, describing the fluid convection and temperature

$$\frac{dx(t)}{dt} = \sigma(y(t) - x(t))$$

$$\frac{dy(t)}{dt} = -x(t)z(t) + rx(t) - y(t)$$

$$\frac{dz(t)}{dt} = x(t)y(t) - bz(t)$$



Edward Lorenz (1917-2008)

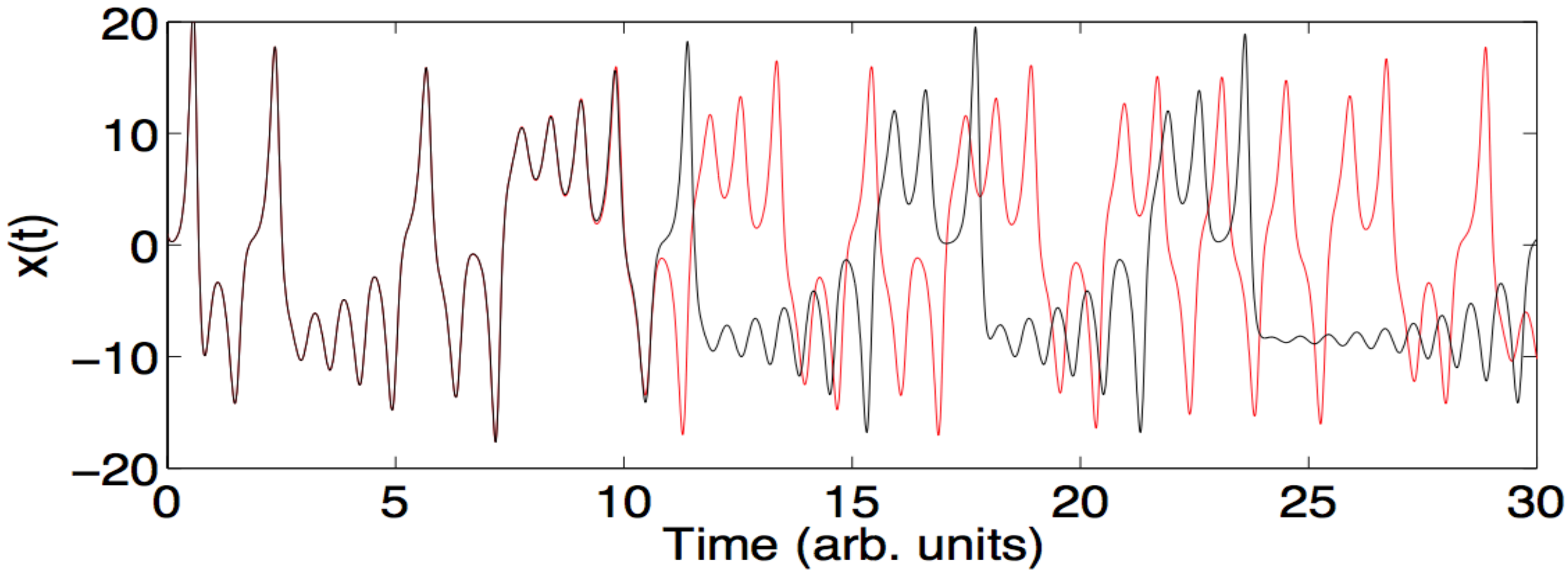
(x,y,z) symbolizes a state of the atmosphere

The Lorenz's model has three degrees of freedom and enough nonlinearity to generate chaotic dynamical behavior

E. Lorenz, Deterministic nonperiodic flow, J of the Atmospheric Science, vol. 20, p. 130 (1963)

The Lorenz model

- The Lorenz's model shows a remarkable feature, **the sensitive dependence on initial conditions** that result in the trajectories in phase space diverging exponentially fast away from each other



$[X_0,Y_0,Z_0] = [0,1,1.05]$

$[X_0,Y_0,Z_0] = [0.000001,1.000001,1.05000001]$

E. Lorenz, Deterministic nonperiodic flow, J of the Atmospheric Science, vol. 20, p. 130 (1963)

The Lorenz model

- A sad ending? In view of the inevitable inaccuracy and incompleteness of weather observations, precise very long-range forecasting would seem to be non-existent

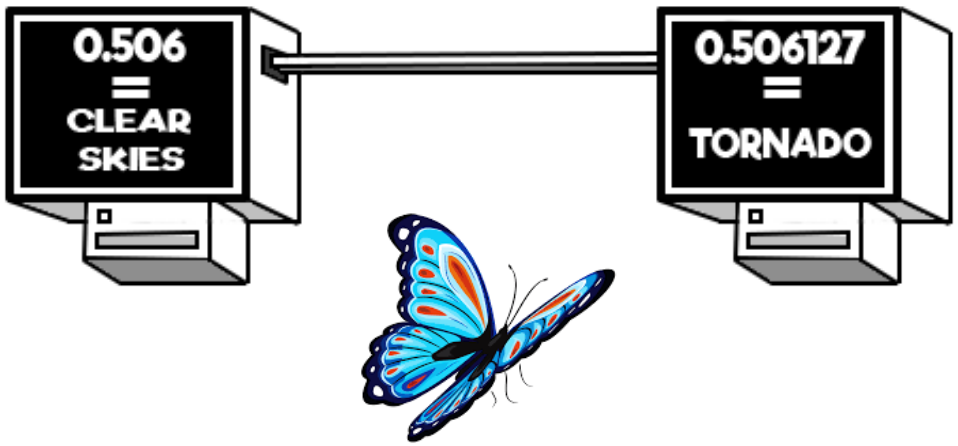
Does the flap of a butterfly's wings in Brazil

set off a tornado in Texas?

Edward Lorentz

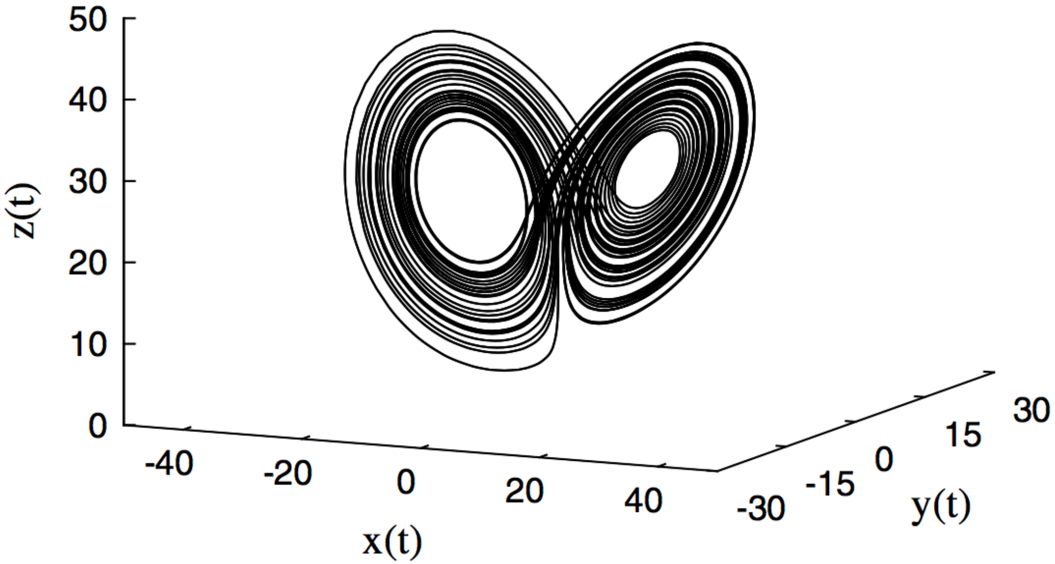
Paper presented at a conference

Washington D.C. 1972



The Lorenz model

- A strange (chaotic) attractor consists on fractal structure, where multiscale similarity of the density of trajectories exists. Here the chaotic attractor looks like a butterfly, and it is called the butterfly attractor



Two points on the attractor that are near each other at one time will be arbitrarily far apart at later times. Strange attractors are unique in that the motion of the system never repeats (non-periodic)

E. Lorenz, Deterministic nonperiodic flow, J of the Atmospheric Science, vol. 20, p. 130 (1963)

Three fingerprints of chaos

- Trajectory is never periodic, but coasts along an imaginary surface called a strange attractor
- Extreme sensitivity to initial conditions
- Sensitive dependence on initial conditions is equivalent to stating that nearby trajectories will diverge exponentially e.g. continuous systems in a 2-dimensional phase space cannot experience such a divergence

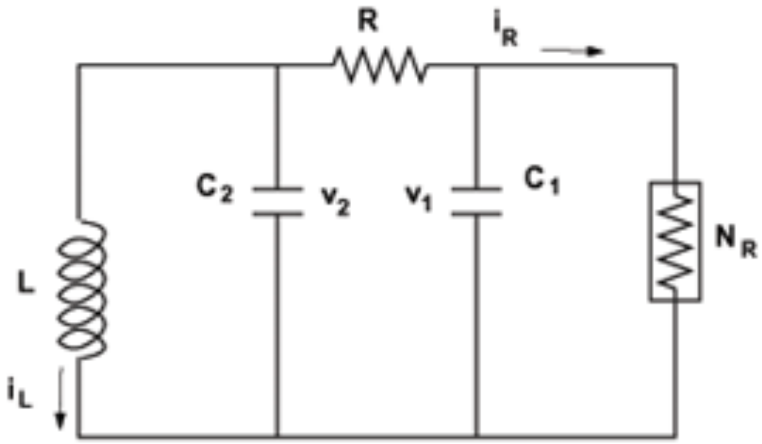
Poincaré-Bendixon theorem (!)

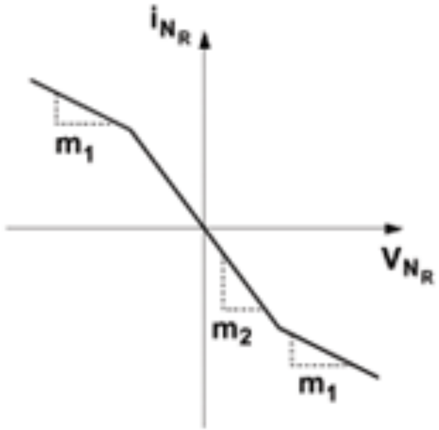
Chaotic behaviors can only be observed in deterministic continuous systems with a phase space of dimension 3, at least

S. Strogatz, "Nonlinear Dynamics and Chaos with application to physics, biology, chemistry and engineering", Perseus Book (1994)

The Chua's circuit, the birth of chaos-based engineering applications

- In 1983, a Berkeley professor, Leon Chua, proposed an electronic circuit exhibiting rich complex nonlinear dynamics including chaos



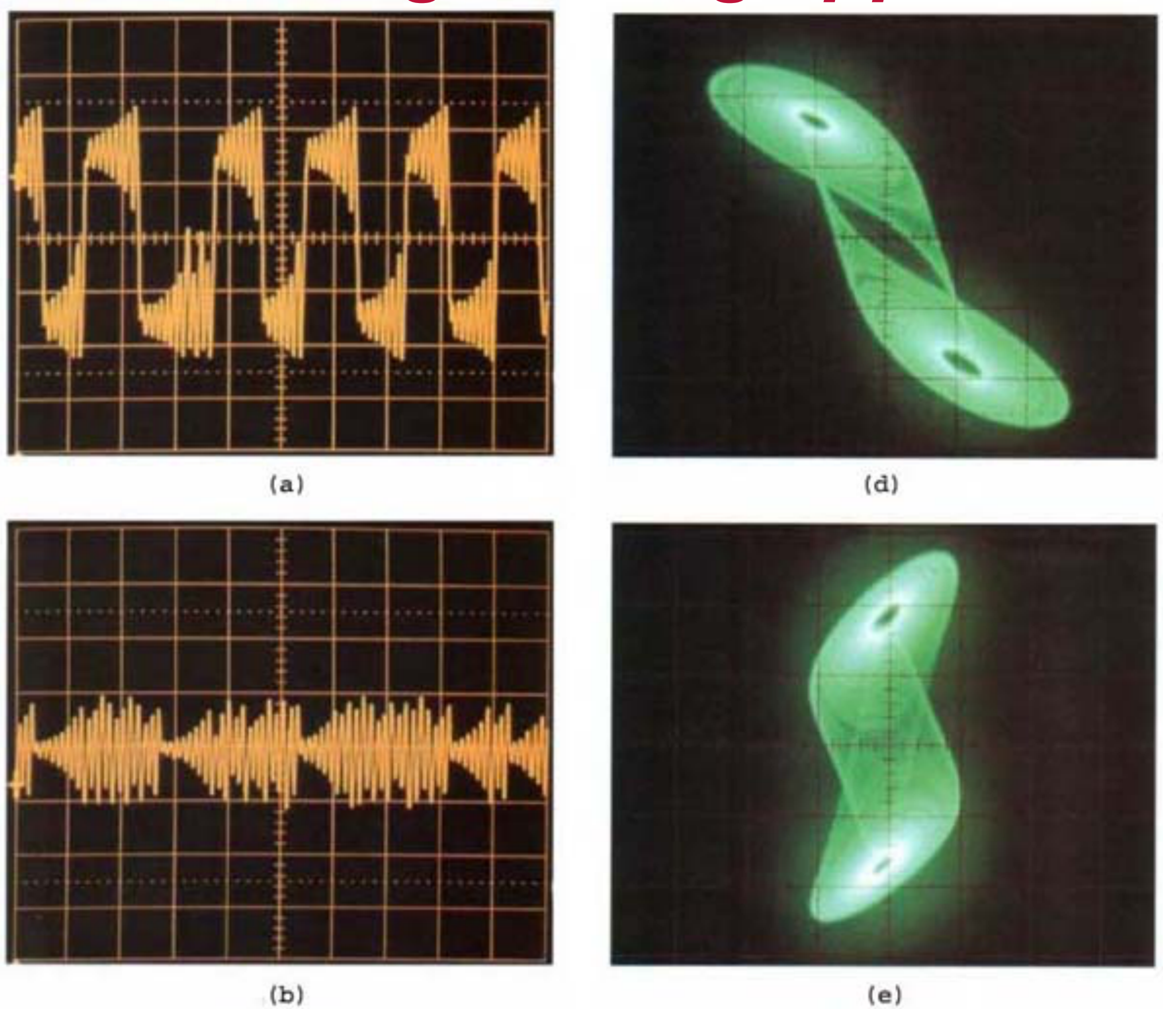


Leon Chua
(1936-)

$$
\begin{aligned}
C_1 \frac{dv_1}{dt} &= \frac{(v_2 - v_1)}{R} - h(v_1)\,, \\
C_2 \frac{dv_2}{dt} &= \frac{(v_1 - v_2)}{R} + i_L\,, \\
L \frac{di_L}{dt} &= -v_2\,,
\end{aligned}
$$

L. Chua et al., IEEE Transactions on Circuits and Systems vol. 33, p. 1072 (1986)

The Chua's circuit, the birth of chaos-based engineering applications



The Chua's circuit exhibiting a double-scroll attractor named as the Chua's attractor is the first experimental proof of chaos (1986)

The first laser instabilities

PHYSICAL REVIEW VOLUME 123, NUMBER 4 AUGUST 15, 1961

Stimulated Optical Emission in Fluorescent Solids. II. Spectroscopy and Stimulated Emission in Ruby

T. H. Maiman,* R. H. Hoskins,* I. J. D'Haenens, C. K. Asawa, and V. Evtuhov
Hughes Research Laboratories, A Division of Hughes Aircraft Company, Malibu, California

(Received January 27, 1961)

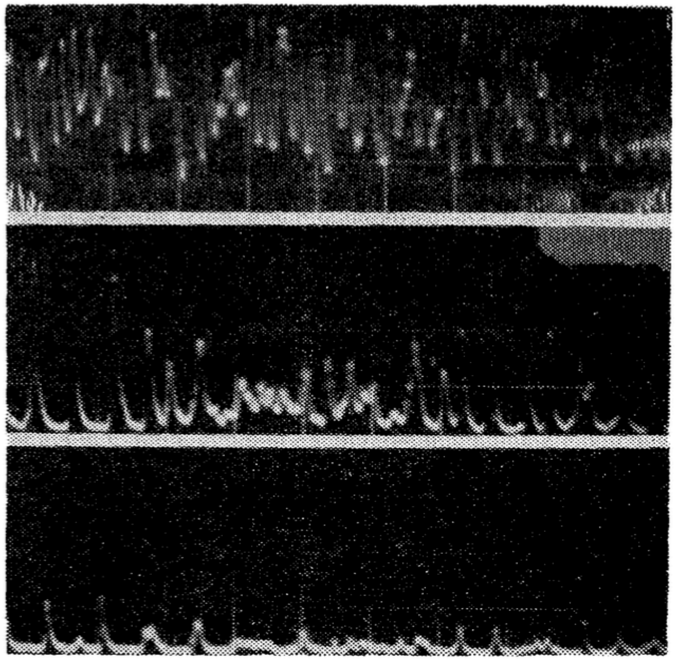


Fig. 13. Output pulse from ruby on an expanded time scale (10 μsec/division). (a), (b), and (c) represent the output approximately 600, 1000, and 1200 μsec after the onset of oscillation. The vertical sensitivity and the base line are the same in each case.

These initial observations were either left unexplained or wrongly attributed to noise!

T. H. Maiman et al., Phys. Rev., vol. 123, pp. 1151 (1961)

Analogy between fluids and lasers

Lorenz

$$\frac{dx(t)}{dt} = \sigma(y(t) - x(t))$$

$$\frac{dy(t)}{dt} = -x(t)z(t) + rx(t) - y(t)$$

$$\frac{dz(t)}{dt} = x(t)y(t) - bz(t)$$

Lorenz-Haken

$$\frac{dx(t)}{dt} = \sigma(y(t) - x(t))$$

$$\frac{dy(t)}{dt} = -x(t)z(t) + rx(t) - (1 - i\delta)y(t)$$

$$\frac{dz(t)}{dt} = \mathrm{Re}(x^*(t)y(t)) - bz(t)$$

- Lorentz: x(t), y(t), and z(t) represent properties of the convecting fluid flow, and temperature differences between the left- and right-hand sides of the cell of fluid and between its top and bottom, respectively
- Lorenz-Haken: x(t), y(t), and z(t) represent the time evolution of the scaled electric field, the atomic polarization, and the population inversion

H. Haken, Physics Letters A vol. 53, p. 77 (1975)

Laser classification

- Number of variables can be reduced by adiabatic elimination
- Need to know the timescale of each variable with respect to each other

	Variables to describe dynamics	Examples of lasers
Class A	Electric field (photons)	He-Ne lasers (632.8 nm) Dye lasers
Class B	Electric field (photons) Population inversion (atoms)	Semiconductor lasers Solid-state lasers CO_2 lasers
Class C	Electric field (photons) Population inversion (atoms) Polarization (matter)	He-Ne lasers (3.39 μm) He-Xe lasers (3.51 μm) NH_3 lasers

Fortunato Tito Arecchi (1933-)

F. T. Arecchi et al., Opt. Comm., vol. 51, pp. 308 (1984)

Laser classification

- 3 dynamical variables with same timescales hence 3 equations are required to describe the laser's dynamic

$$\kappa_c \approx \gamma_\perp \approx \gamma_{||}$$

$$\frac{dE(t)}{dt} = -\kappa(E(t) + AP(t))$$

$$\frac{dP(t)}{dt} = -P(t) - E(t)D(t)$$

$$\frac{dD(t)}{dt} = \gamma(1 - D(t) + E(t)P(t))$$

K_c Electric field decay rate
$\gamma_{//}$ Population inversion decay rate
$\gamma_\perp$ Atomic polarization decay rate

$$\kappa = \kappa_c/\gamma_\perp \quad \gamma = \gamma_{||}/\gamma_\perp$$

Class C lasers satisfy the necessary condition for generating chaos i.e., at least three independent variables are necessary for chaos, and chaotic behaviors are easily observed

F. T. Arecchi et al., Opt. Comm., vol. 51, pp. 308 (1984)

Laser classification

- Atomic polarization much faster e.g. 2 dynamical variables required

$$\gamma_{\perp} \gg \kappa_c > \gamma_{||}$$

- Laser's dynamic is described with two equations (field and population)

$$\frac{dE(t)}{dt} = \kappa(-1 + AD(t))E(t)$$

$$\frac{dD(t)}{dt} = \gamma\left(1 - D(t) - E^2(t)D(t)\right)$$

K_c Electric field decay rate
$\gamma_{//}$ Population inversion decay rate
$\gamma_{\perp}$ Atomic polarization decay rate

Class B lasers do not satisfy the condition for generation of chaos. They are stable in nature. The phase space is 2-dimensional and allows fixed points and periodic trajectories term limit cycles

F. T. Arecchi et al., Opt. Comm., vol. 51, pp. 308 (1984)

Laser classification

- Both atomic polarization and population inversion relax very fast e.g. 1 dynamical variable is required

$$\gamma_\perp \approx \gamma_{||} \gg \kappa_c$$

- Laser's dynamic can be described with only one equation (electric field)

$$\frac{dE(t)}{dt} = \kappa\left(-1 + \frac{A}{1+E^2(t)}\right)E(t)$$

$$\approx \kappa\left(-1 + A - AE^2(t)\right)E(t)$$

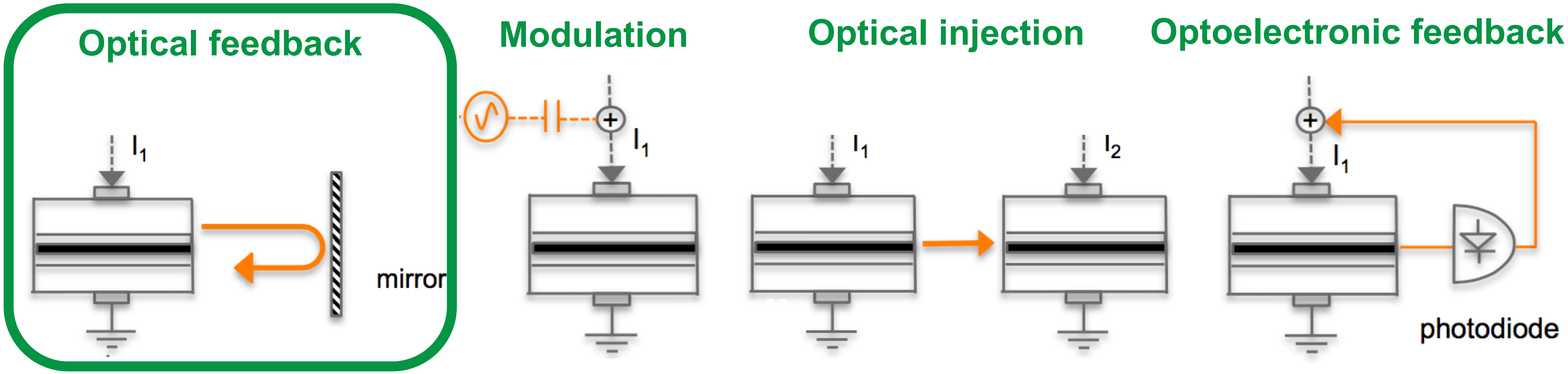
K_c Electric field decay rate
$\gamma_{//}$ Population inversion decay rate
$\gamma_\perp$ Atomic polarization decay rate

Class A lasers are the most stable lasers, however, they may show chaotic behaviors by external perturbations with two or more extra degrees of freedom, as in the case of class B lasers

F. T. Arecchi et al., Opt. Comm., vol. 51, pp. 308 (1984)

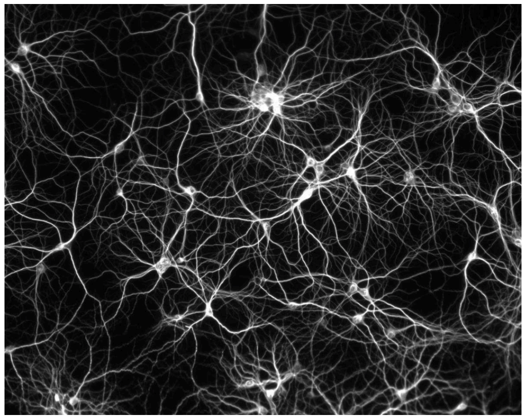
Chaos generation in diode lasers

- Adding external perturbations allow to fulfill the Poincaré-Bendixon and a minimum of 3 degrees of freedom is at least obtained



- Some other optical feedback systems

Traffic



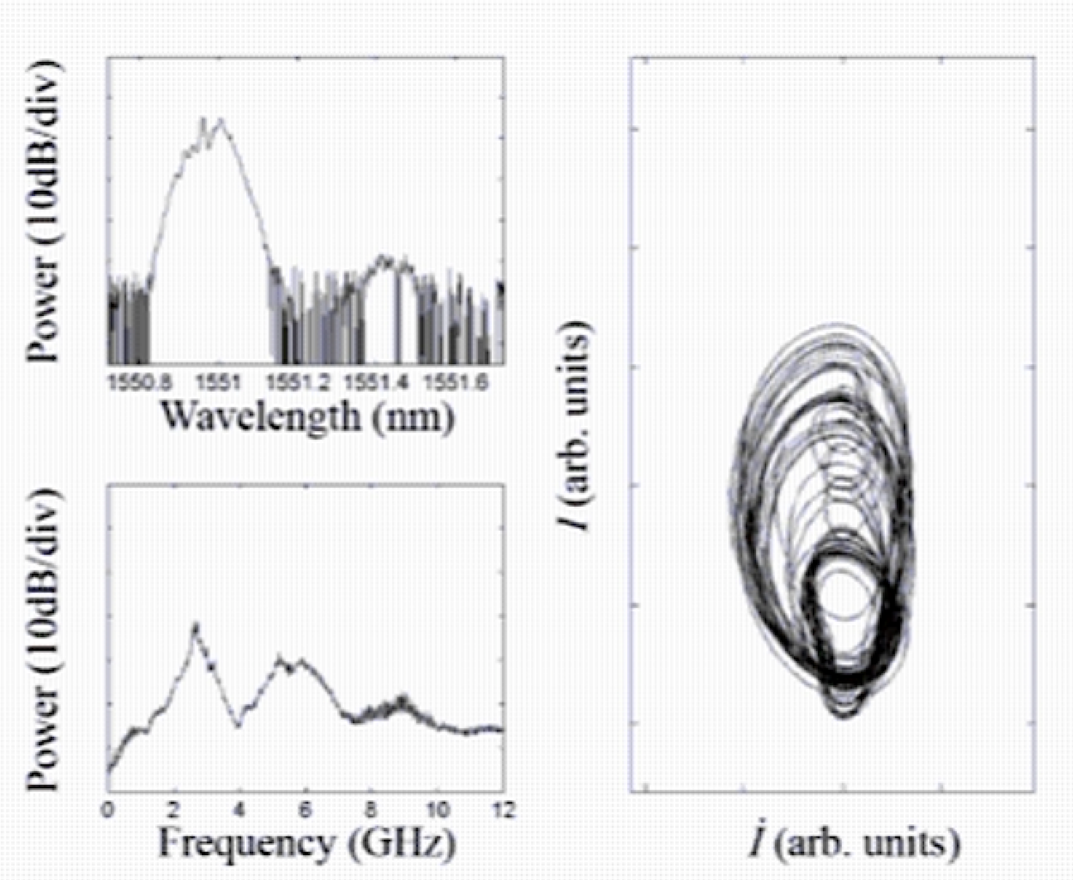
Neurons

Chaos-based applications

- Private communications, light detection and ranging, random bit generation
- Manifestation of chaos : complex dynamics in the optical & electrical domains; chaos attractor in the phase-space (I, dI/dt) with I: photon density

Large optical spectrum

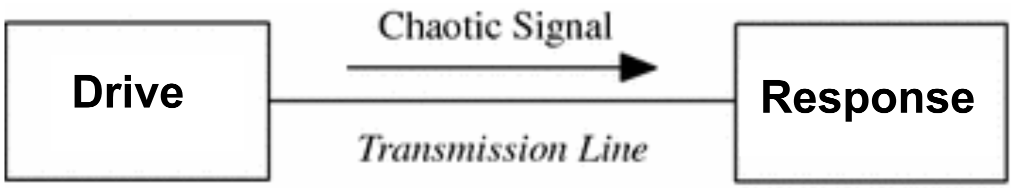
Large electrical spectrum



Complex phase space

Chaos synchronization

- Drive & response lasers synchronize themselves



Awase-Gai

VOLUME 64, NUMBER 8 PHYSICAL REVIEW LETTERS 19 FEBRUARY 1990

Synchronization in Chaotic Systems

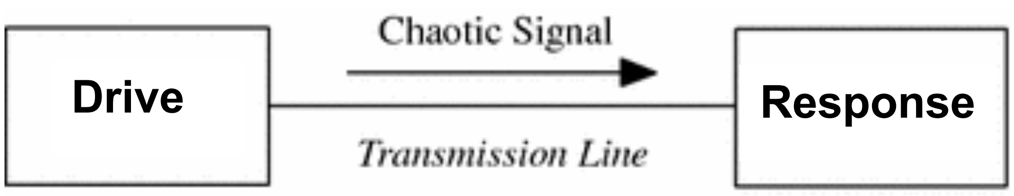
Louis M. Pecora and Thomas L. Carroll
Code 6341, Naval Research Laboratory, Washington, D.C. 20375
(Received 20 December 1989)

- A very elegant mathematical proof of the synchronization can be conducted by extracting the Lyapunov function associated to the error function

R. He and P. G. Vaidya, Phys. Rev. E. vol. 57, p. 1532 (1998)

Chaos synchronization

- Identical lasers leads to identical synchronization



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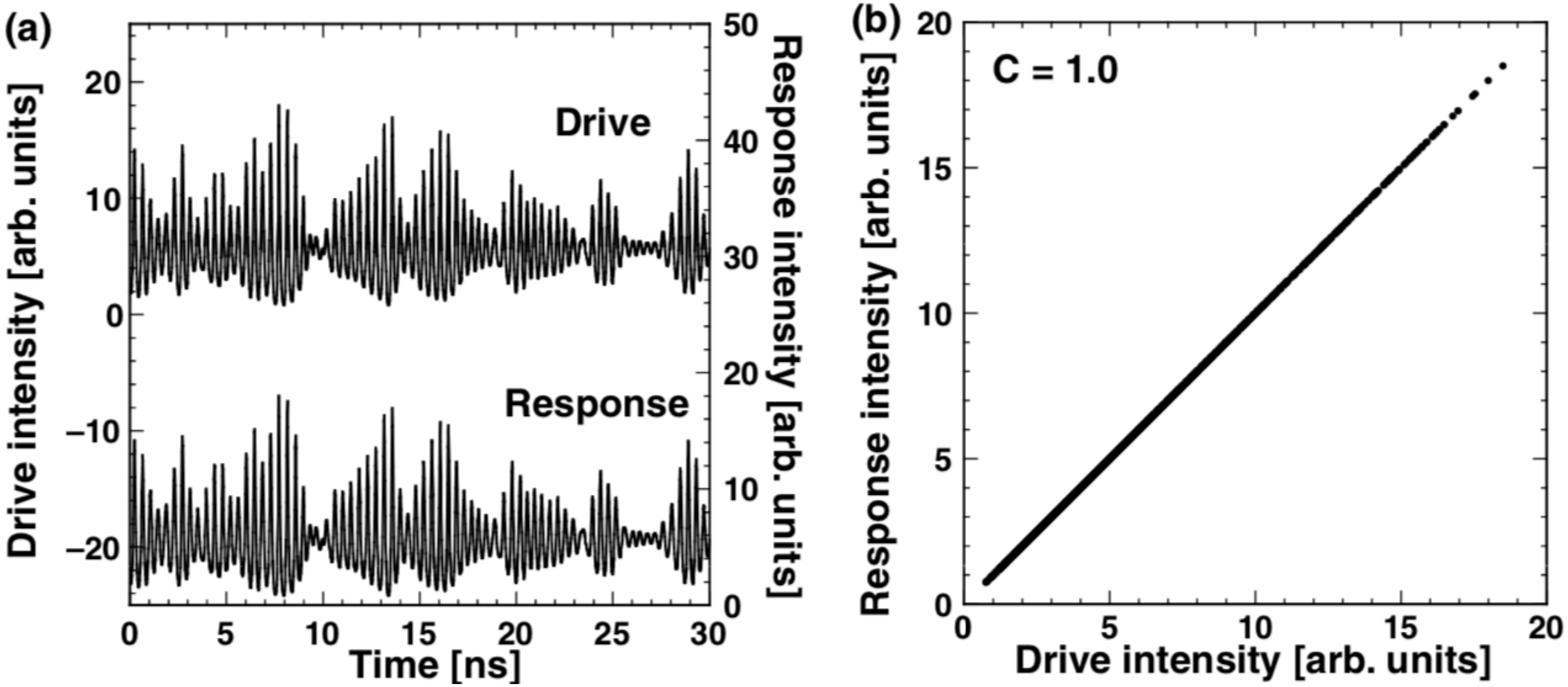
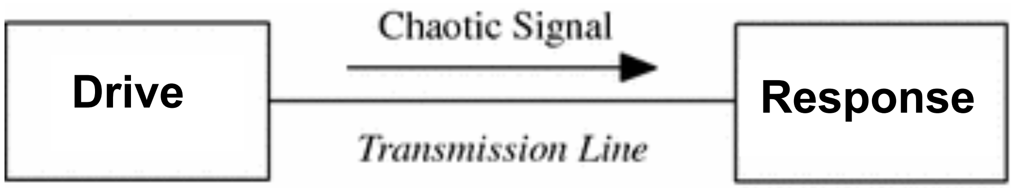


Chaos synchronization

- Non-identical lasers: the synchronization is good but not identical



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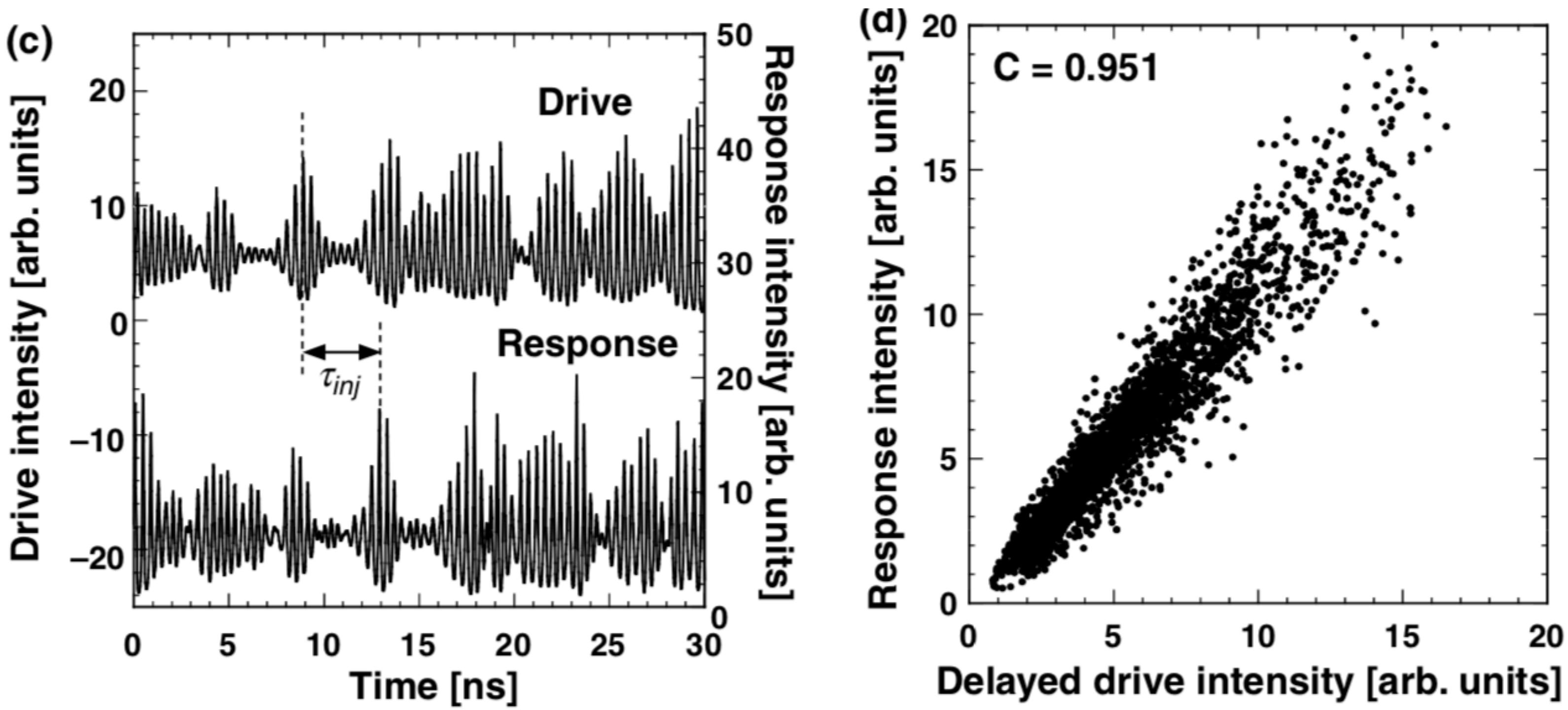
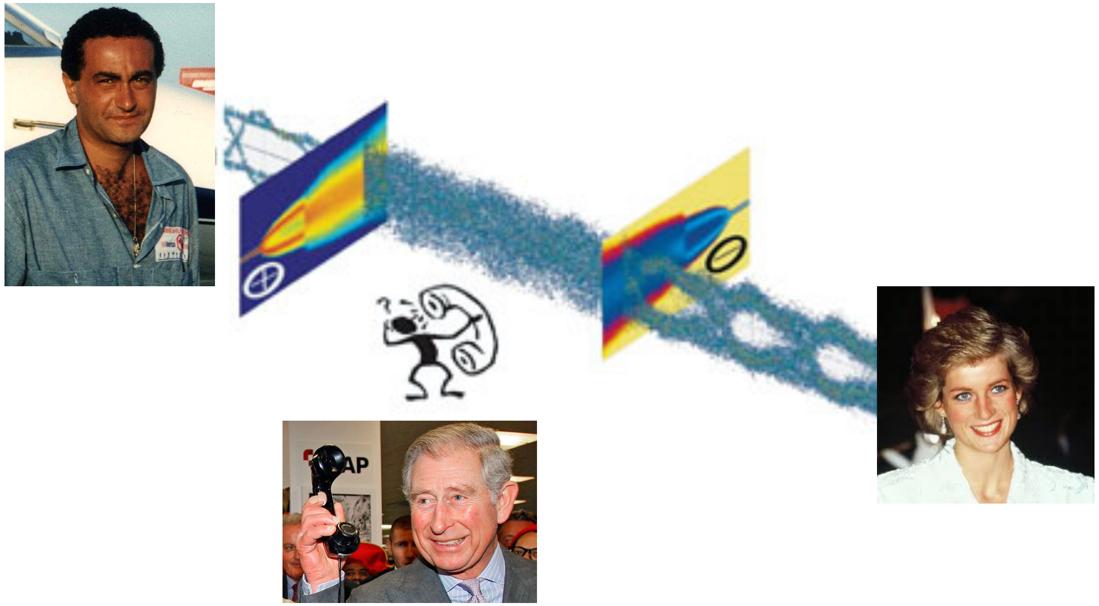
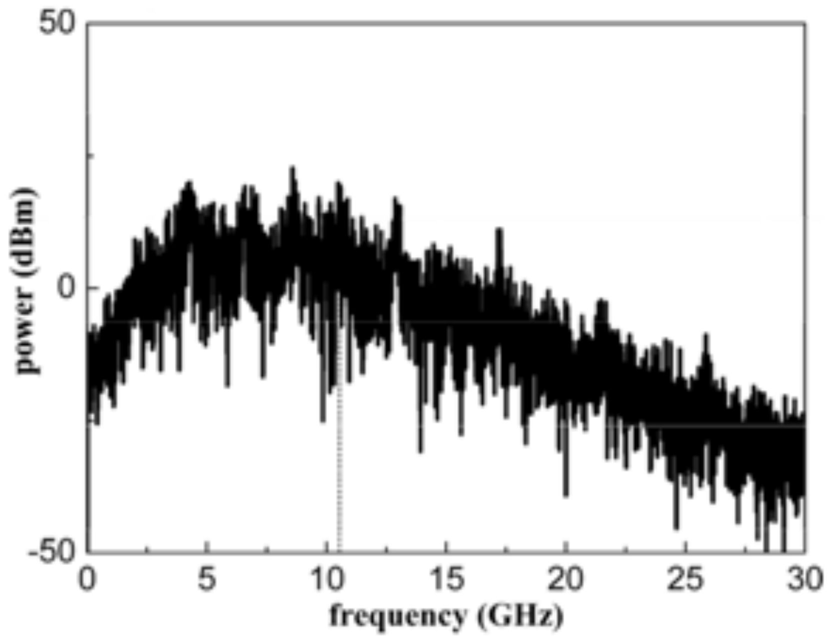


Private communications

- The broadband chaotic signal can be used for increasing privacy



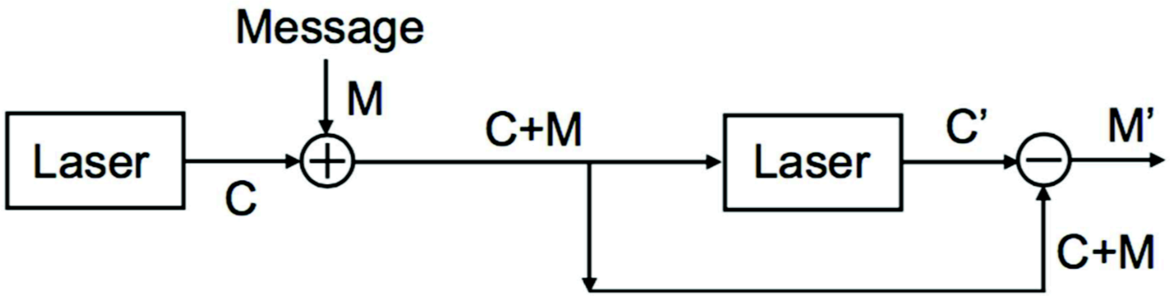


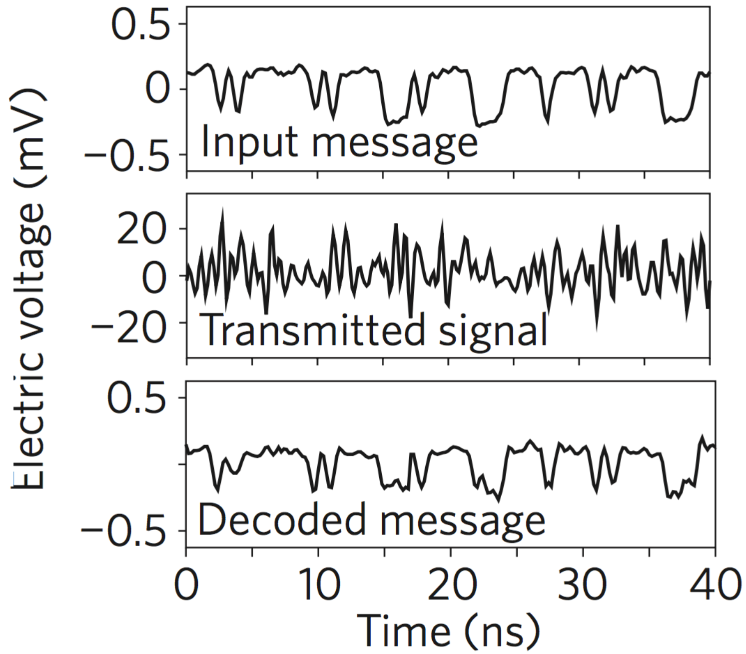
The generation of the chaos is based on the input signal, so only the receiver that has knowledge on how the chaos is generated can reproduce the chaos and cancel it to recover the signal

A. Uchida, Optical Communication with Chaotic Lasers, Wiley, (2012)

Private communications

- Diode lasers allow fiber optic chaos communication systems with high data rates (oscillation frequency)
- Possible extension to atmospheric free-space communications using quantum cascade lasers

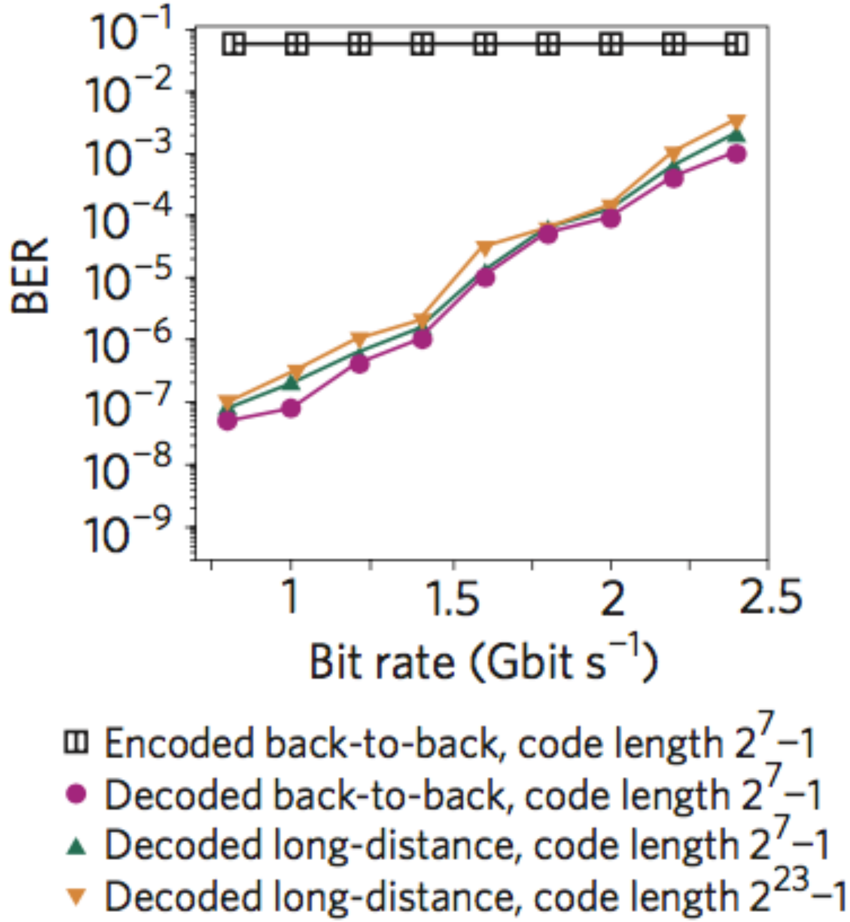


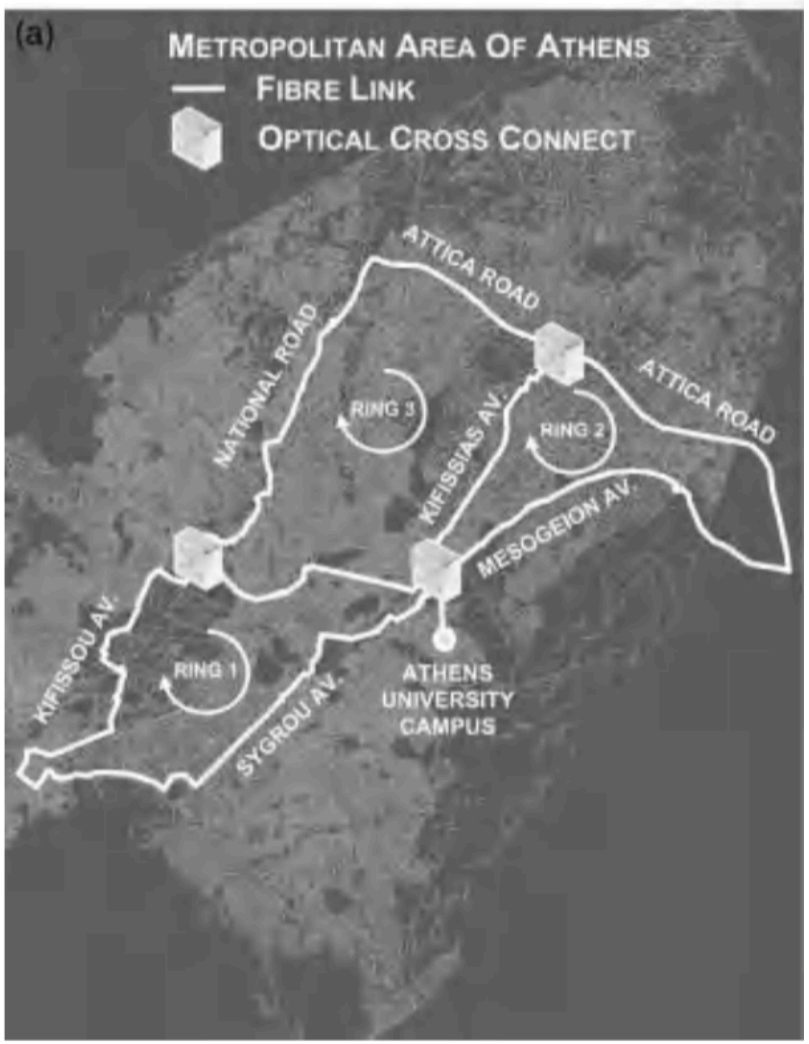


By spreading the narrow band signal into a wideband signal, chaos-based communications can both create desired jamming and avoid malicious jamming

Private communications

- The optical chaos-covered signal was transmitted over a 120 km distance in a commercial optical network over the metropolitan area of Athens

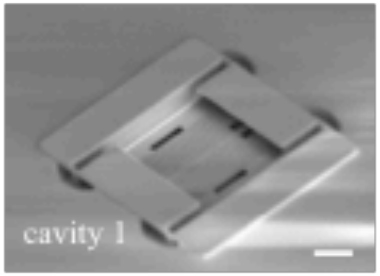


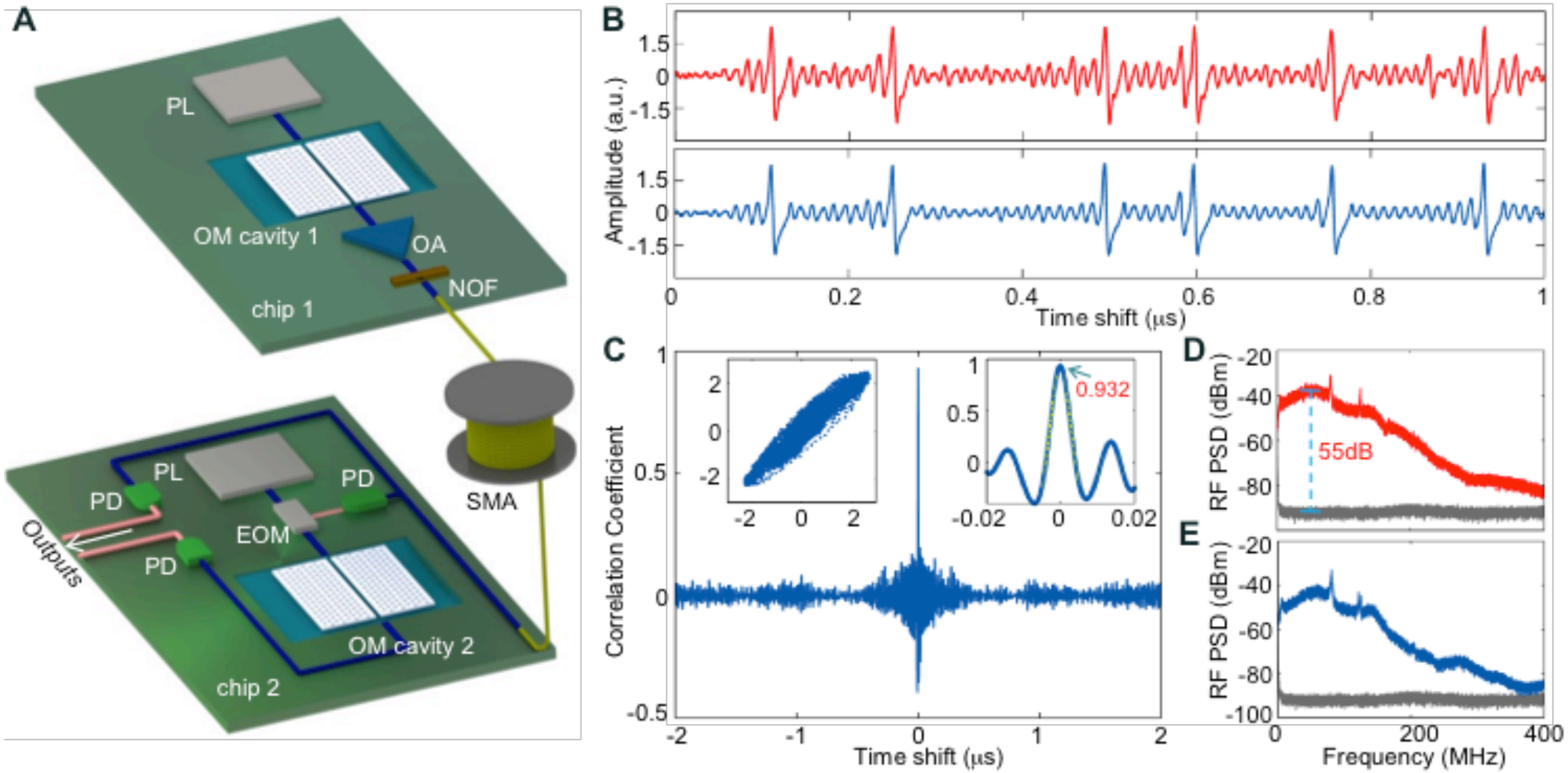


Argyris, A. et al., Nature vol. 438, pp. 343–346 (2005)

On-chip private communications

- Optomechanical chaos with silicon microcavities
  - Made with photonics crystals
  - Integrable on a photonics circuit



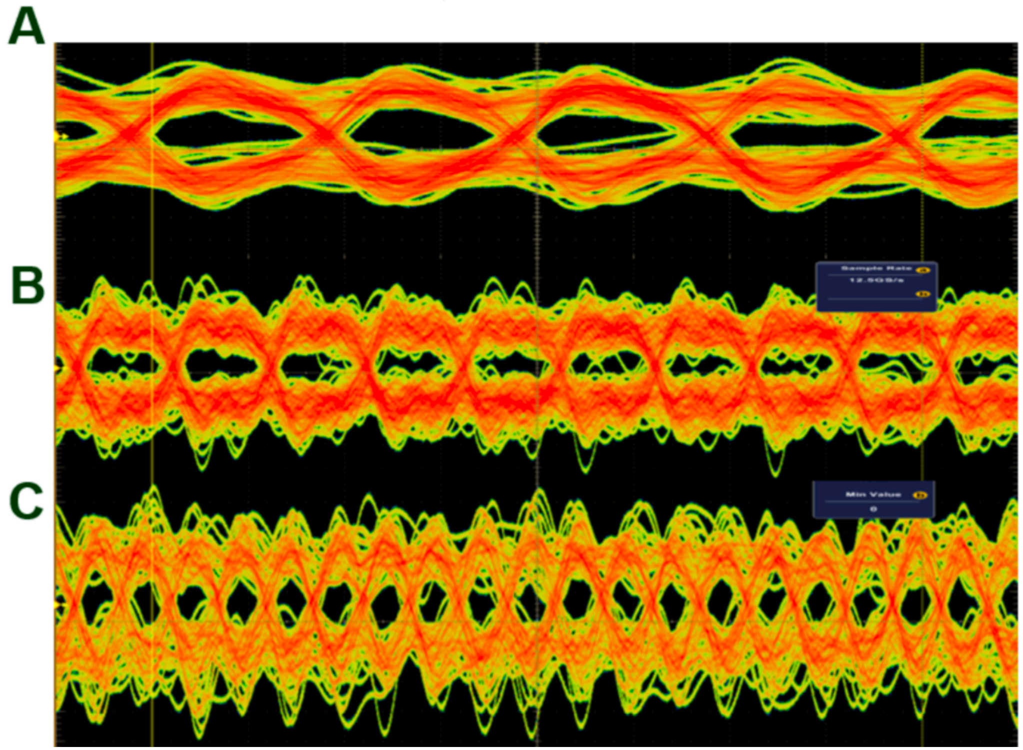


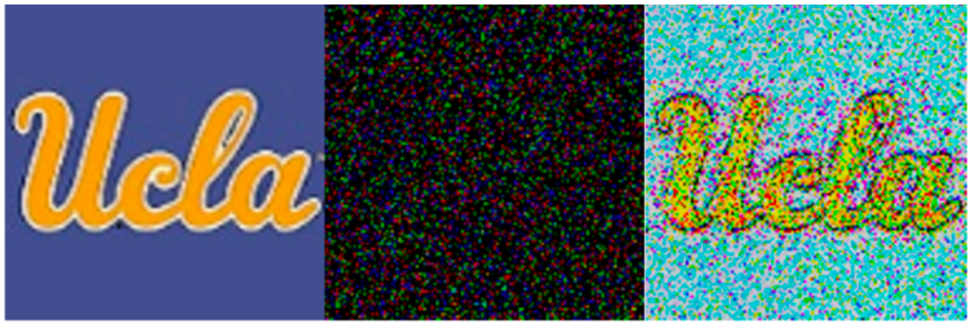
On-chip private communications

Eye diagram of decrypted
0.5 Mbit/s digital message

Eye diagram of decrypted
1 Mbit/s digital message

Eye diagram of decrypted
2 Mbit/s digital message





Encrypted Decrypted

Test image with the logo of
University of California Los Angeles
(UCLA)

Private communications

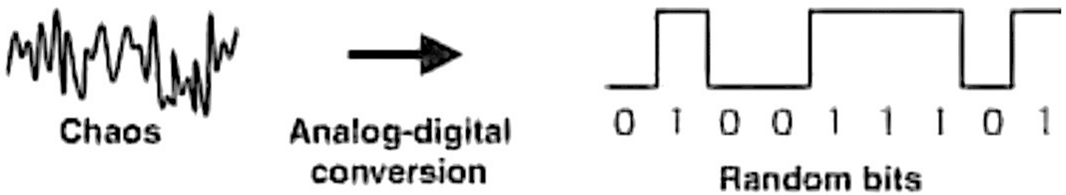
- Privacy in chaos communications results from the fact that an eavesdropper must have the proper hardware and parameter settings in order to decode and recover the message
- In conventional encryption techniques, a key is used to alter the symbols used for conveying information. Tx/Rx share a key so that the information can be recovered
- The key in chaos communication is a set of parameter values as well as similar hardware structure that are required for chaos synchronization and cancelation. The key is static even if a time-varying chaotic carrier is used

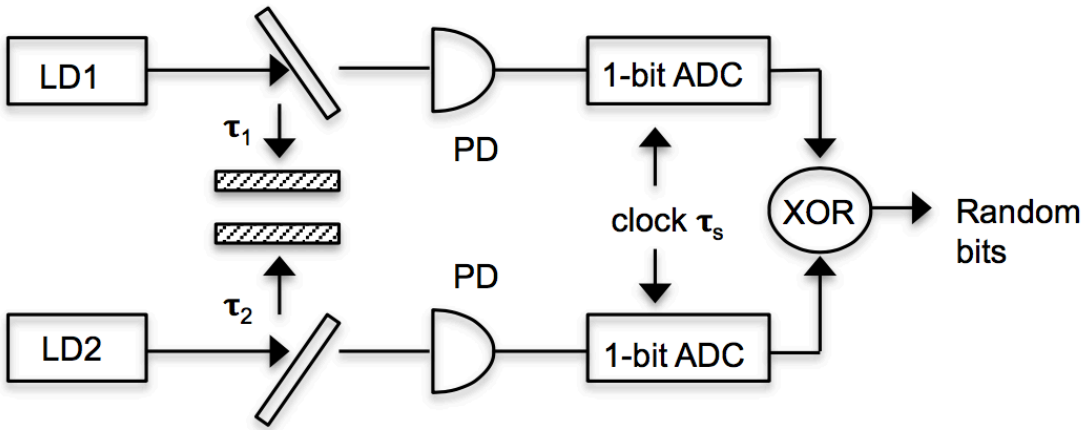
The two important factors for privacy considerations in chaos communication are the dimensionality of the chaos and the effort required to obtain the necessary parameters for a matched Rx

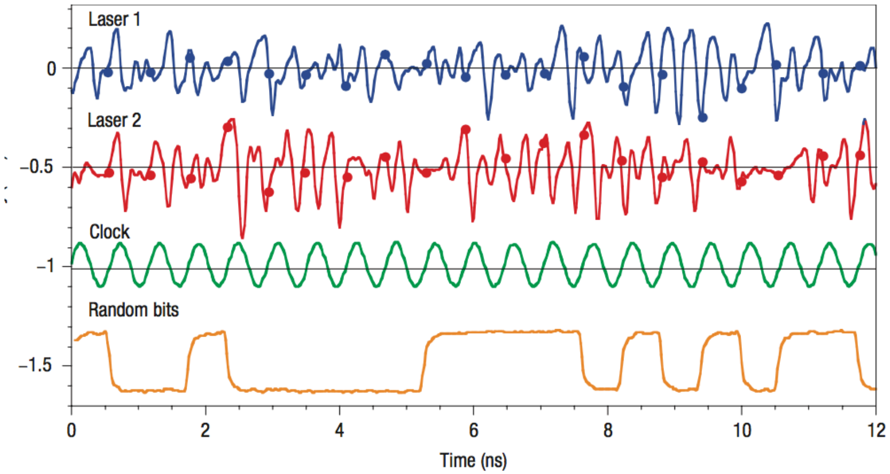
Y. Hong et al., Power loss resilience and eavesdropper detection in optical chaos communications systems, SPIE Photonics Europe. Vol. 7720, 77200J-1 (2010)

Randomness, extraction, distillation

- A chaotic signal of laser output is detected by a photo-detector and converted to a binary digital signal y an analog-to-digital converter (ADC)
- The ADC converts the input analog signal into a binary digital signal by comparing with the threshold voltage



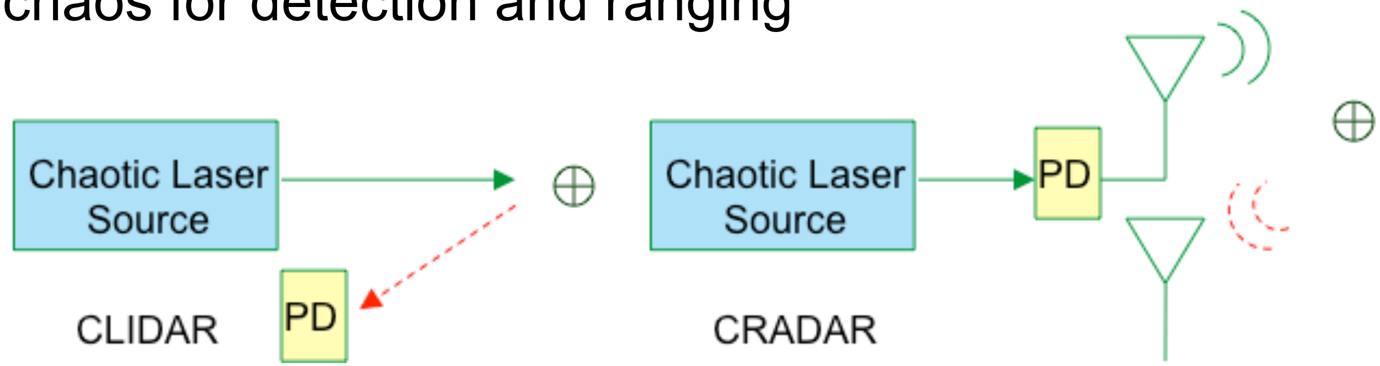




M. Sciamanna and K. A. Shore, Nature Photonics, Vol. 9, p. 151 (2015)

Detection and ranging

- Optical chaos for detection and ranging



- Chaotic lidar (CLIDAR) is a lidar system that uses chaotic light directly as the light source for detections

- Chaotic radar (CRADAR) is a radar system that uses chaotic microwave waveforms converted from chaotic light radiating and receiving by antennas for detections

Their sources are the same → chaotic laser

Their principles are similar → correlation

F.-Y. Lin and J.-M. Liu, IEEE J. Sel. Top. Quantum Electron., vol. 10, pp. 991, (2004)

Detection and ranging

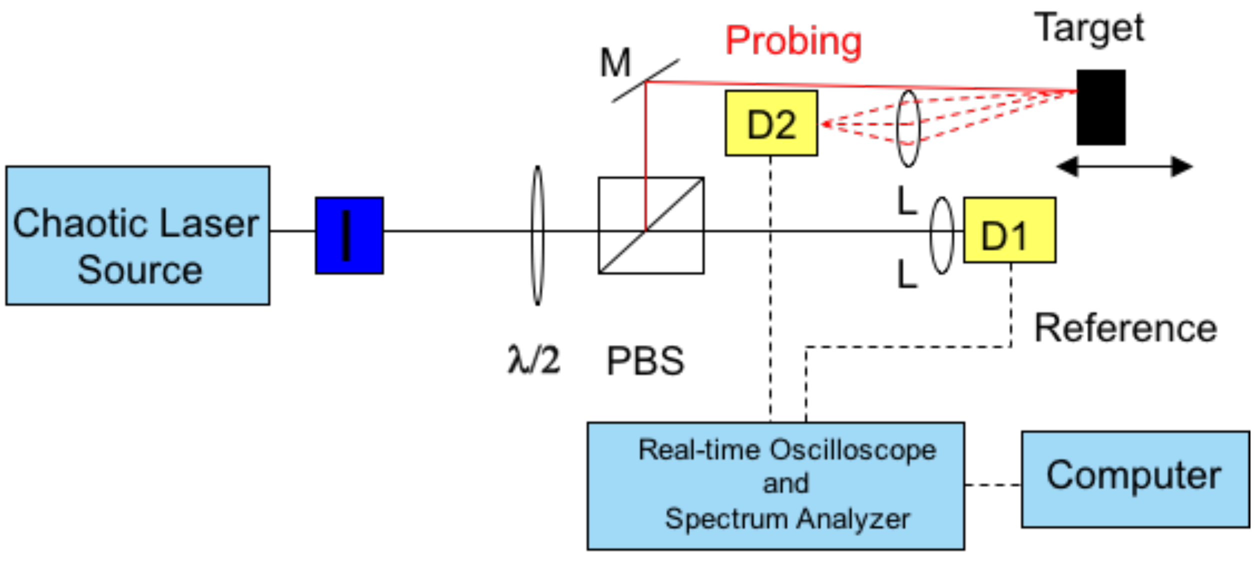
- **Applications of Lidars**
  - range finding
  - traffic safety monitoring
  - target identification and recognition
  - wind/speed detection
  - atmosphere monitoring
  - gas/aerosol composition/distribution detection
  - pollution detection
  - fire/volcano monitoring
  - Self-driving cars

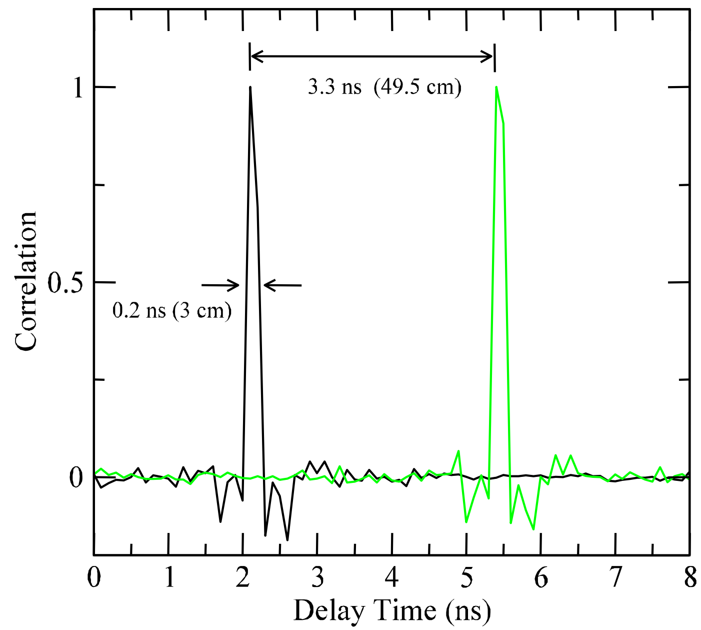
	CLIDAR
SNR	Low
Size	**Compact**
Weight	**Portable**
Cost	**Cheap**
Maintenance	**Easy**
Eye-safety	**Safer**
Efficiency	**High**
Electronics needed	**None (chaos is self-generated by semiconductor lasers)**
Detection range	**Unlimited**
Resolution	**Excellent**

F.-Y. Lin et al., Optics Express, vol. 26, p. 12230, (2018)

Detection and ranging

- Detection is performed by correlating the signal waveform reflected or backscattered from the target with a delayed reference waveform
- 1 cm resolution for targets at 2 m distance
- Resistant to noise and jamming





F.-Y. Lin and J.-M. Liu, IEEE J. Sel. Top. Quantum Electron., vol. 10, pp. 991, (2004)
F.-Y. Lin et al., Optics Express, vol. 26, p. 12230, (2018)